- Površinska temperatura Sunca iznosi približno 5700 K, a zvijezde Sjevernjače 8300 K. Pretpostavivši da se površine ovih dviju zvijezda ponašaju poput crnih tijela, odredi na kojim valnim duljinama spektralne distribucije zračenja imaju maksimume? U kojem području spektra najviše zrači Sunce, a u kojem Sjevernjača (područje X-zraka, UV, vidljivi dio, infracrveni...)? Odredi i snagu kojom ove zvijezde zrače po 1 cm^2 svoje površine.

#

$T_{Sunca} = 5700\,K$

$T_{Sjevernjača} = 8300\,K$

$\lambda_{max}^{Sunce} = ?$, $\lambda_{max}^{Sjevernjače} = ?$ koji dio spektra? $\longrightarrow$ WIEN-ov zakon

$$\boxed{\lambda_{max} \cdot T = 2.898 \cdot 10^{-3}\, m\,K}$$

$P^{Sunce} = ?$, $P^{Sjevernjače} = ?$ po $1\,cm^2 = 10^{-4}\,m^2$

$\longrightarrow$ STEFAN-BOLTZMAN-ov zakon

$$\boxed{P = \sigma_{SB}\, T^4}$$ snaga po jedinici površine ($1\,m^2$)

$$\sigma_{SB} = 5.67 \cdot 10^{-8}\, W\,m^{-2}\,K^{-4}$$

$$\lambda_{max}^{Sunce} = \frac{2.898 \cdot 10^{-3}\, m\,\cancel{K}}{5700\,\cancel{K}} = 508.4 \cdot 10^{-9}\, m \approx 508\, nm$$ $\rightarrow$ područje vidljive svjetlosti (400-700 nm)

$$\lambda_{max}^{Sjev.} = \frac{2.898 \cdot 10^{-3}\, m\,\cancel{K}}{8300\,\cancel{K}} = 349.1 \cdot 10^{-9}\, m = 349\, nm$$

$\longrightarrow$ UV područje (< 400 nm)

$$P_{Sunce} = 5.67 \cdot 10^{-8}\, W\, m^{-2}\, \cancel{K^{-4}} \cdot (5700\,\cancel{K})^4$$

$$= 5.985 \cdot 10^{15} \cdot 10^{-8}\, \frac{W}{m^2} = 5985 \cdot 10^{4}\, \frac{W}{10^4\, cm^2} = 5985\, \frac{W}{cm^2}$$

$$P_{Sjev.} = 5.67 \cdot 10^{-8}\, W\, \overbrace{m^{-2}}^{10^{-4}\,cm^{-2}}\, \cancel{K^{-4}}\, (8300\,\cancel{K})^4 \approx 26909\, \frac{W}{cm^2}$$

- Slobodni elektron se giba duž x osi. Mjerenje njegove brzine duž te osi dalo je rezultat $1.88 \cdot 10^6$ m/s. Preciznost mjerenja je bila 1%.

  a) Kolika je neodređenost u položaju ovog elektrona? (Naputak: uvjerite se prvo u opravdanost korištenje nerelativističke aproksimacije.)

  b) Odredite njegovu de Broglieovu valnu duljinu i njenu neodređenost.

  c) Izrazite kinetičku energiju elektrona u eV-ima, kao i njenu neodređenost. Usporedite kinetičku energiju s energijom mirovanje elektrona.

4

$v = 1.88 \cdot 10^6\ m/s$

$\frac{\Delta v}{v} = 1\% = 0.01$

a) $\Delta x = ?$

b) $\lambda,\ \Delta\lambda = ?$

c) $E,\ \Delta E = ?$

$\frac{v}{c} = \frac{1.88 \cdot 10^6}{3 \cdot 108} = 0.0063 \ll 1$

$\Downarrow$

nerelativistička aproksimacija ok

$p = m_e v$

$dp = m_e\, dv \Rightarrow \Delta p = m_e \Delta v$

$\Delta p = m_e\, 0.01 \cdot v = 0.01\, p$

a) HEISENBERG-ova relacija neodređenosti

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \geq \frac{\hbar}{2}\frac{1}{\Delta p} = \frac{\hbar}{2}\frac{1}{m_e \Delta v} = \frac{\hbar}{2}\frac{1}{0.01 p}$$

$$m_e = 9.1 \cdot 10^{-31} kg = 0.510 \cdot 10^6 eV/c^2$$

$$p = m_e v = 0.510 \cdot 10^6 \frac{eV}{c^2} \cdot 0.0063\, c = 3.2 \cdot 10^3 eV/c$$

$$= 9.1 \cdot 10^{-31} kg \cdot 1.88 \cdot 10^6 m/s$$
$$= 1.71 \cdot 10^{-24}\ kg\, m/s$$

$$\Delta x \geq \frac{hc}{4\pi}\frac{1}{0.01 pc} = \frac{1.240 \cdot 10^{-6}\ \cancel{eV} m}{4\pi \cdot \underbrace{0.01}_{10^{-2}} \cdot 3.06 \cdot 10^3\ \cancel{eV}} = 3.1 \cdot 10^{-9} m$$

$$\Delta x \geq \frac{1.054 \cdot 10^{-34}\ \overset{kg\frac{m^2}{s^2}s}{Js}}{2}\ \frac{1}{10^{-2} \cdot 1.71 \cdot 10^{-24}\ kg\, m/s} = 3.1 \cdot 10^{-9} m$$

b) de BROGLIE-ova relacija

$$p = \frac{h}{\lambda} \qquad dp = -\frac{h}{\lambda^2} d\lambda \quad \Rightarrow \Delta p = \left| -\frac{h}{\lambda^2} \right| \Delta\lambda$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1{,}240 \cdot 10^{-6}\, eV\, m}{3.2 \cdot 10^{3}\, eV} = 0{,}388 \cdot 10^{-9}\, m$$

$$= 0{,}388\ nm$$

$$= \underline{\underline{3{,}88\ \text{Å}}} \quad (\text{x-rays})$$

$$\Delta\lambda = \Delta p \frac{\lambda^2}{h} = 0.01pc \frac{\lambda^2}{hc} = \frac{0.01 \cdot 3.2 \cdot 10^3 eV}{1.240 \cdot 10^{-6} eV\, m} (3.88 \cdot 10^{-10})^2 m^2$$

$$= \underline{\underline{3{,}885 \cdot 10^{-12}\, m}}$$

ali

$$\frac{\Delta\lambda}{\lambda} = \frac{3.89 \cdot 10^{-12} m}{3.88 \cdot 10^{-10} m} \approx 10^{-2} m$$

ili

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta p \frac{\lambda^2}{h}}{\frac{h}{p}} = \Delta p \frac{\frac{\frac{h^2}{p^2}}{h}}{\frac{h}{p}} = \Delta p \frac{\frac{h^2}{p^2} \cdot p}{h^2} = \frac{\Delta p}{p}$$

$$= \frac{\Delta v}{v} = 0.01$$

$$\Delta\lambda = 0.01\lambda = 3.88 \cdot 10^{-12} m$$

c)

$$E = m_e c^2 + E_k \qquad m_e c^2 = 0{,}510\ MeV$$

$$E_k = \frac{m_e v^2}{2} = \frac{m_e c^2}{2} \cdot \frac{v^2}{c^2} = \frac{0{,}510}{2} (0.063)^2\ MeV$$

$$= 10^{-5} \cdot 10^6 eV = \underline{\underline{10\, eV}}$$

$$E = \left(0{,}510 + 0{,}510 \frac{0.063^2}{2}\right) MeV = (0{,}510 + 0.00001)\ MeV$$

$$= 0.51001\ MeV$$

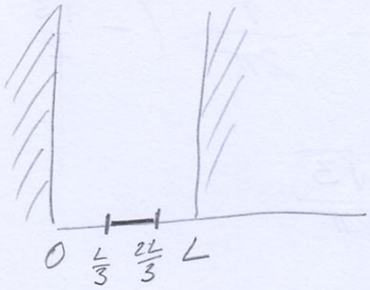
$$dE_k = \frac{m}{2} 2v\,dv = mv\,dv \quad \Rightarrow \quad \Delta E_k = m v \Delta v$$

$$\frac{\Delta E_k}{E_k} = \frac{mv}{\frac{mv^2}{2}} \Delta v = 2\frac{\Delta v}{v} = 0.02 \quad \Rightarrow \Delta E_k = 0.02\, E = \underline{\underline{0{,}2\, eV}}$$

$E_k \ll m_e c^2$ i.e. $10\, eV \ll 0{,}51 \cdot 10^6 eV$

- Elektron je zarobljen u beskonačnoj jednodimenzionalnoj potencijalnoj jami širine L i nalazi se u osnovnom kvantnom stanju. Koji postotak ukupnog vremena elektron provede u srednjoj trećini jame?

#

besk. pot. jama $\longrightarrow$ 

osnovno stanje $\longrightarrow n=1$

postotak vremena $\Leftrightarrow$ vjerojatnost pronalaženja ?

od $x=\frac{L}{3}$ do $x=\frac{2L}{3}$

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}\quad,\quad n=1,2,3\quad;\quad \psi_n(x,t)=\psi_n(x)e^{-i\frac{E_n}{\hbar}t}$$

$P(x,t)\,dx=\psi^*(x,t)\,\psi(x,t)\,dx$ ... vjerojatnost da se u trenutku t čestica nađe u intervalu $(x, x+dx)$

$\Downarrow$

$$P_n(x,t)\,dx=\psi_n^*(x)\cancel{e^{i\frac{E_n}{\hbar}t}}\,\psi_n(x)\cancel{e^{-i\frac{E_n}{\hbar}t}}\,dx$$

$$\int_{L/3}^{2L/3}P_1(x,t)\,dx=\left|\psi_1(x)=\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}\right|=$$

$$=\int_{L/3}^{2L/3}\left(\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}\right)^2dx=$$

$$=\frac{2}{L}\int_{L/3}^{2L/3}\sin^2\overset{a}{\frac{\pi}{L}}x\,dx=\left|\begin{array}{l}\text{BRONŠTAJN, str. 427, 275}\\ \int\sin^2ax\,dx=\frac{1}{2}x-\frac{1}{4a}\sin 2ax\end{array}\right|$$

$$=\frac{2}{L}\left[\frac{1}{2}x-\frac{1}{4(\pi/L)}\sin\left(2\frac{\pi}{L}x\right)\right]\Bigg|_{L/3}^{2L/3}$$

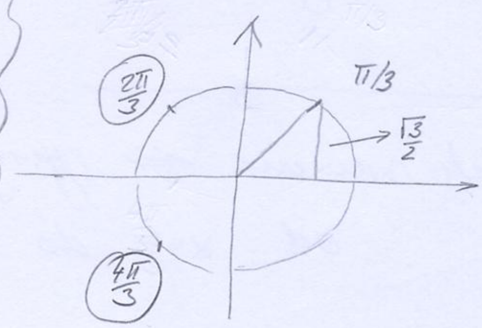
$$=\frac{2}{L}\left[\frac{1}{2}\left(\frac{2L}{3}-\frac{L}{3}\right)-\frac{L}{4\pi}\left(\sin\frac{2\pi}{\cancel{L}}\frac{2\cancel{L}}{3}-\sin\frac{2\pi}{\cancel{L}}\frac{\cancel{L}}{3}\right)\right]$$

$$=\frac{2}{L}\left[\frac{1}{6}L-\frac{L}{4\pi}\left(\sin\frac{4\pi}{3}-\sin\frac{2\pi}{3}\right)\right]=$$

$$= \frac{1}{3} - \frac{1}{2\pi}\Big(\underbrace{-\frac{\sqrt{3}}{2}}_{\sin 4\pi/3} - \underbrace{\frac{\sqrt{3}}{2}}_{\sin 2\pi/3}\Big) =$$

$$= \frac{1}{3} + \frac{\sqrt{3}}{2\pi}$$

$$= 0.609 = 60.9\%$$



$\Downarrow$

Elektron ~61% vremena provede u srednjoj trećini jame

lako provjeriti

$$\int_0^{L/3} P_1(x,t)\,dx = \int_{2L/3}^{L} P_1(x,t)\,dx = \frac{1}{3} - \frac{1}{2\pi}\frac{\sqrt{3}}{2}$$

$$\Downarrow$$

$$\int_0^{L} P_1(x,t)\,dx = \int_0^{L/3} P_1(x,t)\,dx + \int_{L/3}^{2L/3} P_1(x,t)\,dx + \int_{2L/3}^{L} P_1(x,t)\,dx = 1 \checkmark$$

- Mjerenje na nekom kvantnom sistemu je našlo iznos impulsa vrtnje $L = 2.584 \cdot 10^{-34}$ J s.

  a) Navedite sve moguće vrijednosti orbitalnih i azimutalnih kvantnih brojeva (l i m). (Naputak: prisjetite se oblika i raspona mogućih svojstvenih vrijednosti L^2 i L_z.)

  b) Koliko raznih kuteva može $\vec{L}$ zatvarati s z-osi? Navedite ih.

  c) Navedite svojstvene funkcije impulsa vrtnje koje odgovaraju kvantnim brojevima iz a) dijela zadatka. Normalizirajte kutno ovisnu valnu funkciju $\varphi(\theta, \phi)$ proporcionalnu jednostavnom zbroju tih svojstvenih funkcija. (Naputak: Iskoristite ortonormiranost kuglinih funkcija.)

(#.)

$L = 2.584 \cdot 10^{-34}\,\mathrm{Js}$

a) $l, m = ?$

$$L^2 = \hbar^2 l(l+1) \Rightarrow l(l+1) = \frac{L^2}{\hbar^2} = \frac{2.584^2}{1.055^2} \approx 6 = 2(2+1)$$

$$\underline{\underline{l = 2}}$$

$$m \in \{-l, \dots l\} = \underline{\underline{\{-2, -1, 0, 1, 0, 2\}}}$$

c) $l = 2$

$m \in \{\underline{-2}, -1, 0, 1, \underline{2}\}$

$$Y_l^m(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}\, P_l^m(\cos\theta)\, e^{im\varphi}$$

$$Y_2^2(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\varphi} \qquad Y_2^1(\theta,\varphi) = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{-2i\varphi} \qquad Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{-i\varphi}$$

$$Y_2^0(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1)$$

$$\varphi(\theta,\varphi) = A\left(Y_2^2 + Y_2^1 + Y_2^0 + Y_2^{-1} + Y_2^{-2}\right)(\theta,\varphi)$$

$$\langle \varphi | \varphi \rangle \stackrel{!}{=} 1$$

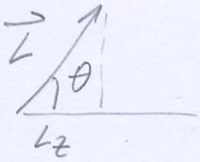
$$A^2 \langle Y_2^2 + Y_2^1 + Y_2^0 + Y_2^{-1} + Y_2^{-2} | Y_2^2 + Y_2^1 + Y_2^0 + Y_2^{-1} + Y_2^{-2}\rangle$$

$= \Big|$ kugline funkcije su ortonormirane $\langle Y_l^m | Y_{l'}^{m'}\rangle = \delta_{ll'}\delta_{mm'} \Big|$

$$= A^2(1 + 1 + 1 + 1 + 1 + 0 + 0 \dots) \stackrel{!}{=} 1$$

$$\Rightarrow A = \underline{\underline{1/\sqrt{5}}}$$

b)



$$\cos\theta = \frac{L_z}{L} = \frac{m\hbar}{\sqrt{l(l+1)}\hbar}$$

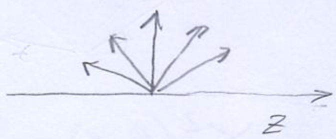
$$\theta_{min} \leftarrow \max(\cos\theta) = \max \frac{m}{\sqrt{l(l+1)}} = \frac{l}{\sqrt{l(l+1)}} = \frac{2}{\sqrt{6}}$$

$$\Rightarrow \theta_{min} = \arccos\frac{2}{\sqrt{6}} = 0,6155 = \underline{\underline{35,26^\circ}}$$

$$\theta(m,l) = \arccos\frac{m}{\sqrt{l(l+1)}}$$

$l=2$

m	θ
2	$35,26^\circ$
1	$\arccos\frac{1}{\sqrt{6}} = 65.90^\circ$
0	$\arccos 0 = 90^\circ$
-1	$\arccos\frac{-1}{\sqrt{6}} = 114,1$
-2	$\arccos -\frac{2}{\sqrt{6}} = 144,74^\circ$



$\downarrow$

5 kuteva