1.

$\underbrace{e^+e^-}_{P\text{ (pozitronij)}}$

$E_3 \longrightarrow E_1$ (γ emisija)

$v_p^0 = 0$

$v_p' = ?$

$$E_n = -\overset{=1^2}{Z^2}\frac{R_p}{n^2} \qquad R_p = R\frac{\mu}{m_e} = 13.6\frac{\mu_p}{m_e}\,eV = 13.6\,\frac{1}{2}\,eV = \underline{6.8\,eV}$$

$$\mu_p = \frac{m_{e^-} m_{e^+}}{m_{e^-} + m_{e^+}} = \underline{\frac{m_e}{2}}$$
(reducirana masa)

$$E_3 = -\frac{6.8}{3^2} = -0.76\,eV$$

$$E_1 = -6.8\,eV$$

$$\Delta E = E_3 - E_1 = (-0.76 + 6.8)\,eV = \underline{\underline{6.04\,eV}}$$

$\ll \underbrace{m_p c^2}_{\text{(masa poz.)}} = 2 m_e c^2 = 1\,MeV$ (*)

ZOI: $\quad 0 = \vec{p}_p' + \vec{p}_\gamma = p_p' - p_\gamma \Rightarrow \boxed{p_\gamma = p_p'}$

ZOE: $\quad \cancel{m_p c^2 +} E_3 = \cancel{m_p c^2 +} \underbrace{\frac{m_p^2 v_p'^2}{2}}_{E_k^p} + \underbrace{h\nu}_{E_\gamma} + E_1$

$$\Delta E = \frac{p_p'^2}{2m_p} + p_\gamma c$$

(*) $\Downarrow$

$\Delta E \ll m_p c^2$

$\Rightarrow$ nerelativistička aproks. ok za E_k^p ✓

$$\Delta E = \frac{p_p'^2}{2m_p} + p_p' c$$

$$\Delta E = \frac{m_p v_p'^2}{2} + m_p v_p' c$$

$$\Rightarrow v_p'^2 + 2c\,v_p' - \frac{2\Delta E}{m_p} = 0$$

$$v_p' = -c \overset{>0}{\oplus} \sqrt{c^2 + \frac{2\Delta E}{m_p}} = c\left(-1 \oplus \sqrt{1 + 2\frac{\Delta E}{m_p}}\right)$$

$$= 5.9\cdot 10^{-6}\,c = \underline{\underline{1770\,m/s}}$$

(2.)

$e^- \rightarrow E_k$, $E = mc^2 + E_k$ (↳ m_e)

$\lambda = \frac{h}{p}$ de Broglieova valna duljina

1, relativistički

$$E^2 = m^2c^4 + p^2c^2$$

$$(mc^2 + E_k)^2 = m^2c^4 + p^2c^2 \Rightarrow \cancel{m^2c^4} + E_k^2 + 2mc^2E_k = \cancel{m^2c^4} + p^2c^2$$

$$\Downarrow$$

$$pc = \sqrt{(mc^2 + E_k)^2 - m^2c^4}$$

$$pc = \sqrt{E_k^2 + 2mc^2E_k} \quad , \quad \frac{p^2}{2m} = E_k + \frac{E_k^2}{2mc^2}$$

2. nerelativistički

$$v \ll c, \quad \frac{v}{c} \ll 1, \quad \boxed{E_k \ll mc^2}$$

$$E_k = \frac{mv^2}{2} = \frac{p^2}{2m} \quad , \quad pc = c\sqrt{2mE_k}$$

$$\lambda_r = \frac{h}{p_r} = \frac{hc}{p_rc} = \frac{hc}{\sqrt{E_k^2 + 2mc^2E_k}}$$

$$\lambda_{nr} = \frac{h}{p_{nr}} = \frac{hc}{p_{nr}c} = \frac{hc}{c\sqrt{2mE_k}} = \frac{hc}{\sqrt{2mc^2E_k}}$$

$$\frac{\lambda_{nr}}{\lambda_r} = \frac{\sqrt{E_k^2 + 2mc^2E_k}}{\sqrt{2mc^2E_k}} = \sqrt{1 + \frac{E_k}{2mc^2}} = 1.05 \quad /^2$$

$$1 + \frac{E_k}{2mc^2} = 1.05^2 \Rightarrow E_k = 2m_ec^2(1.05^2 - 1)$$

$$= 0.205\, m_ec^2$$

$$= 20.5\%\, m_ec^2 \quad (m_ec^2 = 0.511\text{ MeV})$$

$$= \underline{0.1\text{ MeV}}$$

(3.)

$N = 10000$

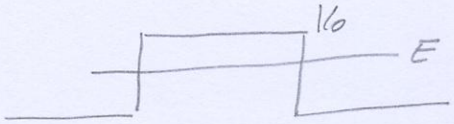
$E = 3\,eV$

$V_0 = 5\,eV$

$a = 0.7\,nm$

$\lambda_{dB} = ?$

$N_T = ?$



$h = 6.63 \cdot 10^{-34}\,Js$

$c = 2.99 \cdot 10^{8}\,m/s$

$1\,eV = 1.6 \cdot 10^{-19}$ (CV) J

$hc = 1.240 \cdot 10^{-6}\,eVm$

$m_e c^2 = 0.51 \cdot 10^{6}\,eV$

$E \ll m_e c^2$

$\Downarrow$

nerelativistički slučaj ✓

$$E = \frac{p^2}{2m_e} \Rightarrow p = \sqrt{2m_e E}$$

$$pc = \sqrt{2m_e c^2 E}$$

$$pc = \sqrt{2 \cdot 0.51 \cdot 10^{6}\,eV \cdot 3\,eV} = 1749.29\,eV$$

$$\lambda_{dB} = \frac{h}{p} = \frac{hc}{pc} = \frac{1.240 \cdot 10^{-6}\,eVm}{1749.29\,eV} = 7.09 \cdot 10^{-10}\,m = 7.09\,\text{Å}$$

$\hbar c = 197.33$ MeV fm

tablice fiz. konst.

$E < V_0$

$$\frac{1}{T} = 1 + \frac{1}{4}\frac{V_0^2}{E(V_0 - E)}\,sh^2(\varkappa a)$$

$$\varkappa = \sqrt{\frac{2m_e(V_0 - E)}{\hbar^2}} = \sqrt{\frac{2m_e c^2 (V_0 - E)}{(\hbar c)^2}} = \sqrt{\frac{2 \cdot 0.51 \cdot 10^{6}\,eV \cdot 2\,eV}{(197.33 \cdot 10^{6}\,eV\,fm)^2}}$$

($fm = 10^{-15}\,m$)

$$= \sqrt{\frac{2 \cdot 0.51 \cdot 2}{197.33^2}} \cdot 10^{24}\,m^{-1} = 0.72 \cdot 10^{10}\,m^{-1}$$

$$T = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2\,sh^2(\varkappa a)}$$

$$= \frac{4 \cdot 3 \cdot 2\,eV^2}{4 \cdot 3 \cdot 2\,eV^2 + 25\,eV^2 \times sh^2(0.72 \cdot 10^{10}\,m^{-1} \cdot 0.7 \cdot 10^{-9}\,m)}$$

$$= \frac{24}{24 + 25\,sh^2(5.04)}$$

$$= 0.00016 = 0.016\,\%$$

$\Rightarrow N_T = 1.6$ elektrona

1–2

4.

vidi rješenje zadatka 3. iz 2. kolokvija 17. lipnja 2016

s / $k \to k-1$

...

a)

$$\langle x(t) \rangle = \bar{x}(t) = \frac{1}{2}\sqrt{\frac{\hbar}{m\omega_0}}\sqrt{\frac{k}{2}}\left(e^{-i\omega_0 t} + e^{i\omega_0 t}\right)$$

$$= \sqrt{\frac{\hbar}{m\omega_0}}\sqrt{\frac{k}{2}}\cos\omega_0 t$$

b) $\psi(x,t)$ <u>nije</u> stacionarno stanje

$\Uparrow$

Dokaz:

$$\rho(x,t) = \psi^*(x,t)\psi(x,t) \neq \rho(x) \quad \text{(tj. ovisi o vremenu)}$$

ili

$$\hat{H}\psi(x,t) \neq \text{cte.}\ \psi(x,t) \qquad (\hat{H}\psi(x,t) = E\psi(x,t) \text{ za stacionarna stanja})$$