- γ-zraka, valne duljine $\lambda = 1\,\mathrm{pm}$ se rasprši na elektronu pod kutem od $60°$. Koliki je Comptonov pomak valne duljine $\Delta\lambda$? Kolika je kinetička energija predana elektronu?

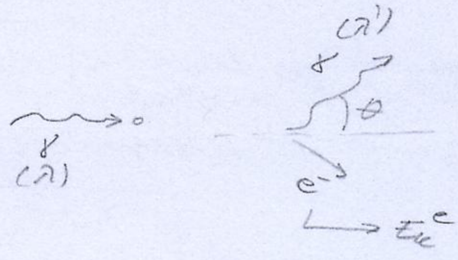
#

$\lambda = 1\,pm = 10^{-12}\,m$

$\theta = 60^\circ$

$\Delta\lambda = \lambda' - \lambda$

$E_k^e = ?$



$$\Delta\lambda = \lambda' - \lambda = \lambda_C(1-\cos\theta) = \frac{h}{m_e c}(1-\cos\theta)$$

$$\frac{h}{m_e c} = 2.43\cdot 10^{-12}\,m$$

COMPTON-ova valna duljina za raspršenje na e^-

$$= 2.43\cdot 10^{-12}\,m\,(1-\underbrace{\cos 60^\circ}_{1/2})$$

$$= \underline{\underline{1.215\cdot 10^{-12}\,m}} = \underline{\underline{1.215\,pm}}$$

ZOE (zakon očuvanja energ.)

$$E_k^e = E_\gamma - E_\gamma' \Leftarrow E_\gamma + (m_e c^2 + 0) = E_\gamma' + (m_e c^2 + E_k^e)$$

$$= h\nu - h\nu'$$

$$= h\frac{c}{\lambda} - h\frac{c}{\lambda'}$$

$$\lambda' = \lambda + \Delta\lambda = 1 + 1.215 = \underline{2.215\,pm}$$

$$= \left(\frac{hc}{1} - \frac{hc}{2{,}215}\right)\cdot 10^{12}\,m^{-1}$$

$$hc = 6.626\cdot 10^{-34}\,J\cancel{s}\cdot 2.998\cdot 10^{8}\,m/\cancel{s}$$

$$= 19.86\cdot 10^{-26}\,J\,m$$

$$= 19.86\cdot 10^{-26}\,\frac{1}{1.602\cdot 10^{-19}}\,eV\,m$$

$$= \underline{12.40\cdot 10^{-7}\,eV\,m}$$

$$= 12.40\cdot 10^{-7}\,eV\cancel{m}\cdot 10^{12}\,\cancel{m^{-1}} \cdot \left(1 - \frac{1}{2.215}\right)$$

$$= 12.40\cdot 10^{5}\,eV\cdot\left(1-\frac{1}{2.215}\right)$$

$$= 6.8\cdot 10^{5}\,eV = \underline{\underline{0.68\,MeV}}$$

$$\{ = 1.09\cdot 10^{-13}\,J$$

- a) Odredite najdužu i najkraću valnu duljinu Lymanove i Paschenove serije (tj. najdužu i najkraću liniju u $m = 1$ i $m = 3$ serijama) u spektru iona Li^{2+} ($Z = 3$) te odredite i ionizacijsku energiju tog iona.

  b) Podsjetimo se da je μ^- (mion) čestica istih svojstava kao e^- samo 207 puta masivnija, a μ^+ je njena antičestica. Na osnovu Bohrovog modela atoma izvedite izraz za energetske razine za egzotične $\mu^+\mu^-$ "atome". Da li će linija koja odgovara prvoj (tj. najdužoj) liniji iz Pfundove serije ($m = 5$ serije) biti u vidljivom dijelu spektra?

(#)

a) Li^{2+} ($z = 3$)

$$E_n = -\frac{Z^2 R}{n^2} \quad \text{za vodik } z = 1$$

$$R = hcR_H = hc\frac{4}{b}, \quad b = 364{,}56\text{ nm} \quad \text{RYDBERG-ova ct.}$$

RYDBERG-ova formula za vodik

$$\frac{1}{\lambda_{mn}} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \Leftarrow h\nu = \frac{hc}{\lambda} = E_n - E_m$$

za atom sa $Z \neq 1$

$$\frac{1}{\lambda_{mn}} = Z^2 R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad m, n \in N, \; n > m$$

$$\left[\lambda_{mn}^{(Z)} = \frac{1}{Z^2 R_H}\frac{m^2 n^2}{n^2 - m^2} = \frac{1}{Z^2}\frac{b}{4}m^2\frac{1}{1 - m^2/n^2}\right]$$

Li^{2+} ($Z = 3$), $\frac{1}{Z^2} = \frac{1}{9}$

91.16 nm

$m = 1$ (Lyman) $\quad \lambda_{1n}^{(3)} = \frac{1}{9}\,\frac{364.56\text{ nm}}{4}\,1\,\frac{n^2}{n^2 - 1}$

$n = 2 \quad \lambda_{12}^{(3)} = \frac{1}{9}\, 91.16\text{ nm}\,\frac{4}{4-1} = \underline{\underline{13.51\text{ nm}}}$

$n = \infty \quad \lambda_{1\infty}^{(3)} = \frac{1}{9}\, 91.16\text{ nm} = \underline{\underline{10.13\text{ nm}}}$

$m = 3$ (Paschen) $\quad \lambda_{3n}^{(3)} = \frac{1}{9}\, 91.16\text{ nm}\quad 9\,\frac{n^2}{n^2 - 9}$

$n = 4 \quad \lambda_{34}^{(3)} = \frac{1}{9}\, 91.16\text{ nm}\; 9\,\frac{16}{16-9} = \underline{\underline{208.37\text{ nm}}}$

$n = \infty \quad \lambda_{3\infty}^{(3)} = \frac{1}{9}\, 91.16\text{ nm}\cdot 9 = \underline{\underline{91.16\text{ nm}}}$

ionizacijska energija:

$$-E_1 = Z^2 R = 9 \cdot 13.6\ eV = \underline{\underline{122.4\ eV}}$$

<u>b)</u> $\mu^- \mu^+$

$$E_n = -\frac{R'}{n^2}$$

$$R' = R\,(m_e \to \mu) = \left| \mu = \frac{m_\mu m_\mu}{m_\mu + m_\mu} = \frac{m_\mu}{2} = \frac{207\, m_e}{2} \right|$$

$$= k_c^2 \frac{(m_\mu/2)\, e^4}{2\hbar^2} = \frac{m_\mu/2}{m_e} R = \underline{\underline{\frac{207}{2} R}}$$

$$\frac{1}{\lambda_{mn}^{\mu^+\mu^-}} = \frac{R'}{hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad m, n \in \mathbb{N} \quad n > m$$

$$\lambda_{mn}^{\mu^+\mu^-} = \frac{1}{R_H'} \frac{m^2 n^2}{n^2 - m^2}$$

$$= \frac{2}{207} \frac{1}{R_H}{}^{b/4}\, m^2 \frac{n^2}{n^2 - m^2}$$

$m = 5$ (Pfund) $$\lambda_{5n}^{\mu^+\mu^-} = \frac{2}{207}\, 91.16\,nm \cdot 25\, \frac{n^2}{n^2 - 25}$$

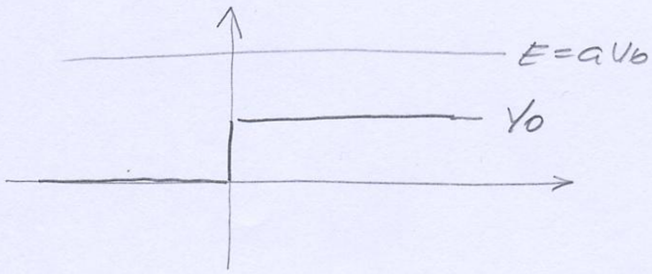
$$n = 6 \qquad \lambda_{56}^{\mu^+\mu^-} = \frac{2}{207}\, 91.16\,nm\; 25\, \frac{36}{36-25} = \underline{\underline{72.06\,nm}}$$

$$\left(n = \infty \qquad \lambda_{5\infty}^{\mu^+\mu^-} = \frac{2}{207}\, 91.16 \cdot 25 = 22.02\,nm \right)$$

nije u vidljivom dijelu spektra (već UV)

- Na jednodimenzionalni potencijalni skok ("step") visine V_0 nalijeće čestica energije aV_0. Za koji a je vjerojatnost refleksije i transmisije ista?

#



$$R = T$$

$$a = ?$$

$E \le V_0 \iff 0 < a \le 1$: $R = 1$, $T = 0$ $\Rightarrow$ $R = T$ ne može biti zadovoljeno

$E > V_0 \iff a > 1$:

$$R = \left(\frac{1 - \frac{\varkappa}{k}}{1 + \frac{\varkappa}{k}}\right)^2, \quad T = 1 - R, \quad \frac{\varkappa}{k} = \sqrt{1 - \frac{V_0}{E}} = \sqrt{1 - \frac{1}{a}}$$

$$R = T \Rightarrow R = 1/2$$

$$\left(\frac{1 - \sqrt{1 - 1/a}}{1 + \sqrt{1 - 1/a}}\right)^2 = \frac{1}{2}$$

$1 - \sqrt{1 - 1/a} > 0$

tj. $\sqrt{1 - 1/a} < 1$ jer $a > 1$

$$\Longrightarrow \quad \frac{1 - \sqrt{1 - 1/a}}{1 + \sqrt{1 - 1/a}} = \frac{1}{\sqrt{2}}$$

$$1 - \sqrt{1 - 1/a} = \frac{\sqrt{2}}{2}\left(1 + \sqrt{1 - 1/a}\right)$$

$$\sqrt{1 - 1/a}\left(1 + \frac{\sqrt{2}}{2}\right) = 1 - \frac{\sqrt{2}}{2}$$

$$\sqrt{1 - 1/a} = \frac{2 - \sqrt{2}}{2 + \sqrt{2}} \quad \Big/^2 \quad \} = 0.1716$$

$$1 - 1/a = \left(\frac{2 - \sqrt{2}}{2 + \sqrt{2}}\right)^2$$

$$\frac{1}{a} = 1 - \frac{(2 - \sqrt{2})^2}{(2 + \sqrt{2})^2} = \frac{(2 + \sqrt{2})^2 - (2 - \sqrt{2})^2}{(2 + \sqrt{2})^2}$$

$$\Rightarrow a = \frac{(2 + \sqrt{2})^2}{8\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{16}(4 + 4\sqrt{2} + 2)\sqrt{2} = \frac{8 + 6\sqrt{2}}{16} = \frac{4 + 3\sqrt{2}}{8} = \underline{\underline{1.03033}}$$

- Za vodikov atom u osnovnom stanju, izračunajte vjerojatnost pronalaženja elektrona između dvije sfere sa polumjerima $r = 1.00\,a_0$ i $r = 1.01\,a_0$, gdje je a_0 Bohrov radijus.

#

$$R_{10}(r) = \left(\frac{1}{a_0}\right)^{3/2} 2 e^{-r/a_0}$$

$$P = \int \Psi^* \Psi \, dV$$

$$P = \int_{1.00 a_0}^{1.01 a_0} dr\, r^2 |R_{10}|^2 \overbrace{\int d\Omega\, Y_{10}^* Y_{10}}^{=1} =$$

$$= \left(\frac{1}{a_0}\right)^3 \cdot 4 \int_{1.00 a_0}^{1.01 a_0} dr\, r^2 e^{-2r/a_0} =$$

BRONŠTAJN, str. 443, rel. 449

$$\int x^2 e^{ax} dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$$

$$a = -\frac{2}{a_0}$$

$$= \frac{4}{a_0^3} \, e^{-\frac{2r}{a_0}} \left(\frac{r^2}{(-2/a_0)} - \frac{2r}{4/a_0^2} + \frac{2}{-8/a_0^3}\right)\Bigg|_{1.00 a_0}^{1.01 a_0}$$

$$= \frac{4}{a_0^3}\Big[e^{-2 \cdot 1.01} \left(-\frac{1.01^2 a_0^2 a_0}{2} - \frac{1.01 a_0 a_0^2}{2} - \frac{a_0^3}{4}\right)$$

$$- e^{-2} \left(-\frac{a_0^2 a_0}{2} - \frac{a_0 a_0^2}{2} - \frac{a_0^3}{4}\right)\Big]$$

$$= 4\Big[e^{-2.02}\left(-\frac{1.01^2}{2} - \frac{1.01}{2} - \frac{1}{4}\right)$$

$$- e^{-2}\left(-\frac{1}{2} - \frac{1}{2} - \frac{1}{4}\right)\Big]$$

$$= 4 \cdot 0.00135331$$

$$= \underline{\underline{0.0054}}$$