- a) Atom apsorbira foton valne duljine 475 nm i odmah potom emitira drugi foton valne duljine 680 nm. Kolika je energija koju atom dobije u ovom procesu? Izrazite energiju u eV.

  b) Pronađite de Broglievu valnu duljinu elektrona ubrzanog iz stanja mirovanja razlikom potencijala od 60 V.

#

a) $\lambda_1 = 475\ nm$ foton

$\lambda_2 = 680\ nm$

$\Delta E = E_1 - E_2 = ?$

$h = 6.626 \cdot 10^{-34}\ Js \approx 6.63 \cdot 10^{-34}\ Js$

$c = 2.998 \cdot 10^{8}\ m = 3 \cdot 10^{8}\ m/s$

foton: $\boxed{E = h\nu = h\frac{c}{\lambda}}$

$(E = pc)$

$hc = 6.63 \cdot 10^{-34}\ J\cancel{s} \cdot 3 \cdot 10^{8}\ m/\cancel{s}$

$= 1{,}99 \cdot 10^{-25}\ Jm$

$= \left| \begin{array}{l} 1eV = 1.602 \cdot 10^{-19}\ CV^{J} \\ \Rightarrow 1J = \frac{1}{1{,}602 \cdot 10^{-19}}\ eV \end{array} \right|$

$= \underline{1{,}24 \cdot 10^{-6}\ eV\ m}$

$\Delta E = E_1 - E_2 =$

$= \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} =$

$= 1.24 \cdot 10^{-6}\ eV\ \cancel{m} \left(\frac{1}{475} - \frac{1}{680}\right) \frac{1}{10^{-9}\ \cancel{m}}$

$= 0.00079 \cdot 10^{3}\ eV$

$= \underline{\underline{0{,}79\ eV}}$

$m_e = 0{,}510 \cdot MeV/c^2$

$m_e c^2 = 0{,}51 \cdot 10^{6}\ eV$

b) $U = 60V$ elektron

$\lambda = ?$

de Broglie-ove rel.

elektron: $\boxed{\nu = \frac{E}{h}, \quad \lambda = \frac{h}{p}}$, $E^2 = m^2c^4 + p^2c^2$

$E = E_k + mc^2$

$E_k = qU = eU = \underline{60\ eV} \ll m_e c^2 = 51 \cdot 10^{4}\ eV \Rightarrow$ ne rel. aproks. je OK $\Big\{ E_k = \frac{p^2}{2m}$

$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\sqrt{E^2 - m_e^2c^4}} = \frac{hc}{\sqrt{(E_k + m_e c^2)^2 - m_e^2 c^4}}$

$= \frac{hc}{\sqrt{\underbrace{E_k^2}_{\ll} + 2E_k m_e c}} \approx \frac{1{,}24 \cdot 10^{-6}\ e\cancel{V}\ m}{\sqrt{2 \cdot 60 \cdot 51 \cdot 10^{4}}\ e\cancel{V}} = 0.0159 \cdot 10^{-6} \cdot 10^{-2}\ m = \underline{\underline{1{,}59 \cdot 10^{-10}\ m}}$ Å

- Vodikov atom prijeđe iz četvrtog pobuđenog ($n = 5$) u osnovno stanje ($n = 1$) i pritom emitira foton. Nakon toga foton se rasprši na slobodnom elektronu (Comptonovo raspršenje) pod kutem od 30°.

   a) Kolika je na kraju valna duljina ovog fotona?

   b) Koju konačnu valnu duljinu bi imao foton da je emitiran iz iona helija He^+ (podsjetnik: helij ima atomski broj 2)?

   c) A iz pozitronija (vezanog sustava e^- i e^+)?

#

a) H atom :

b) He^+ ion :

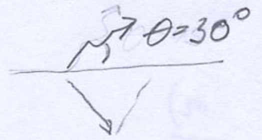
c) e^+e^- tj. pozitronij :

prijelaz

$n = 5 \longrightarrow \underset{\sim}{m} = 1 \;\Rightarrow\; \gamma \longrightarrow \; e^-$

(ν, λ)

COMPTON-ovo raspršenje:



$\lambda'_\gamma = ?$

$$\boxed{E_n - E_m = h\nu = \frac{hc}{\lambda}}$$

$$\boxed{E_n = -Z^2 \frac{R}{n^2}}$$

$\Rightarrow \dfrac{1}{\lambda_{nm}} = \underbrace{R_H}_{\frac{R}{hc}} \left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$, $m, n \in \mathbb{N}$, $n > m$

← formula za emisiju fotona za H atom (Z=1)

za H atom $Z=1$ i

$R = \dfrac{\alpha}{2} c^2 m_e = 13.6\,eV$

općenito je ovdje reducirana masa $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ ali kako je jezgra puno teža od e^- onda $\mu \approx m_e$ za atome

za He^+ $Z=2$,

a za pozitronij $m_e \to \mu = \dfrac{m_e m_e}{m_e + m_e} = \underline{\underline{\dfrac{m_e}{2}}}$, i $Z=1$

⇓

<u>a)</u> $\dfrac{1}{\lambda} = \dfrac{R}{hc}\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right) \Rightarrow \lambda = \dfrac{hc}{R} \dfrac{m^2 n^2}{n^2 - m^2}$

<u>b)</u> $\dfrac{1}{\lambda} = \dfrac{Z^2 R}{hc}\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right) \Rightarrow \lambda = \dfrac{hc}{Z^2 R} \dfrac{m^2 n^2}{n^2 - m^2}$

$Z^2 = 4$

<u>c)</u> $\dfrac{1}{\lambda} = \dfrac{R'}{hc}\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right) \Rightarrow \lambda = 2\dfrac{hc}{R} \dfrac{m^2 n^2}{n^2 - m^2}$

$R' = R\dfrac{\mu'}{m_e} = R\dfrac{m_e/2}{m_e} = \dfrac{R}{2}$

COMPTON-ovo raspršenje

$$\boxed{\lambda' = \lambda + \lambda_C (1 - \cos\theta)}$$

$\lambda_C = 2.43 \cdot 10^{-12}\,m$

a)

$$\lambda = \frac{hc}{R} \underbrace{\frac{m^2 n^2}{n^2 - m^2}}$$

$$\frac{1 \cdot 5^2}{5^2 - 1} = \frac{25}{24}$$

$R = 13.6\,eV$

$h = 6.626 \cdot 10^{-34}\,Js$

$c = 2.998 \cdot 10^{8}\,m/s$

$1\,eV = 1.602 \cdot 10^{-19}\,J$

$$hc = 6.626 \cdot 10^{-34}\,J\cancel{s} \cdot 2.998 \cdot 10^{8}\,m/\cancel{s}$$

$$= \frac{6.626 \cdot 10^{-34} \cdot 2.998 \cdot 10^{8}}{1.602 \cdot 10^{-19}}\,eVm$$

$$= 1.24 \cdot 10^{-6}\,eVm$$

$\Downarrow$

$$\lambda = \frac{1.24 \cdot 10^{-6}\,\cancel{eV}m}{13.6\,\cancel{eV}} \cdot \frac{25}{24}$$

$$= 9.5 \cdot 10^{-8}\,m = 95\,nm$$

$$\lambda' = \lambda + \lambda_C (1 - \cos\theta) =$$

$$= 9.5 \cdot 10^{-8}\,m + 2.43 \cdot 10^{-12}\,m \underbrace{(1 - \overbrace{\cos 30^\circ}^{\sqrt{3}/2})}_{0.134}$$

$$= 9.5 \cdot 10^{-8}\,m + 0.33 \cdot 10^{-12}\,m$$

$$\approx 9.5 \cdot 10^{-8}\,m$$

b)

$$\lambda = \frac{hc}{4R} \frac{m^2 n^2}{n^2 - m^2} = \frac{1}{4} \cdot 9.5 \cdot 10^{-8}\,m = 2.38 \cdot 10^{-8}\,m$$

$$\Rightarrow \lambda' = 2.38 \cdot 10^{-8}\,m + 0.71 \cdot 10^{-12}\,m \simeq 2.38 \cdot 10^{-8}\,m$$

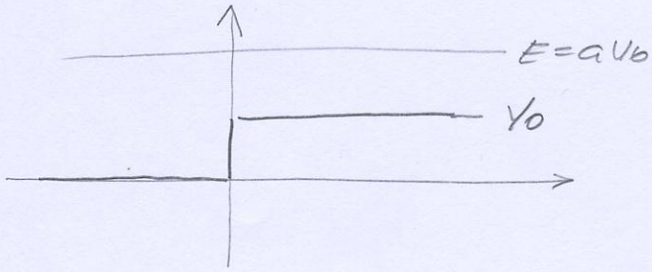
c)

$$\lambda = 2 \frac{hc}{R} \frac{m^2 n^2}{n^2 - m^2} = 2 \cdot 9.5 \cdot 10^{-8}\,m = 19 \cdot 10^{-8}\,m$$

$$\Rightarrow \lambda' = 19 \cdot 10^{-8}\,m + 0.71 \cdot 10^{-12}\,m \simeq 19 \cdot 10^{-8}\,m$$

- Na jednodimenzionalni potencijalni skok ("step") visine V_0 nalijeće čestica energije aV_0. Za koji a je vjerojatnost refleksije i transmisije ista?

#



$$\frac{R = T}{a = ?}$$

$E \leq V_0 \iff 0 < a \leq 1$: $R = 1,\ T = 0 \implies R = T$ ne može biti zadovoljeno

$E > V_0 \iff a > 1$:

$$R = \left(\frac{1 - \frac{\varkappa}{k}}{1 + \frac{\varkappa}{k}} \right)^2, \quad T = 1 - R, \quad \frac{\varkappa}{k} = \sqrt{1 - \frac{V_0}{E}} = \sqrt{1 - \frac{1}{a}}$$

$$R = T \implies R = 1/2$$

$$\left(\frac{1 - \sqrt{1 - 1/a}}{1 + \sqrt{1 - 1/a}} \right)^2 = \frac{1}{2}$$

$1 - \sqrt{1-1/a} > 0$

$$\xrightarrow{} \quad \frac{1 - \sqrt{1 - 1/a}}{1 + \sqrt{1 - 1/a}} = \frac{1}{\sqrt{2}}$$

tj. $\sqrt{1-1/a} < 1$
jer $a > 1$

$$1 - \sqrt{1 - 1/a} = \frac{\sqrt{2}}{2} \left(1 + \sqrt{1 - 1/a} \right)$$

$$\sqrt{1 - 1/a} \left(1 + \frac{\sqrt{2}}{2} \right) = 1 - \frac{\sqrt{2}}{2}$$

$$\sqrt{1 - 1/a} = \frac{2 - \sqrt{2}}{2 + \sqrt{2}} \quad \Big/^2 \quad \} = 0.1716$$

$$1 - 1/a = \left(\frac{2 - \sqrt{2}}{2 + \sqrt{2}} \right)^2$$

$$\frac{1}{a} = 1 - \frac{(2 - \sqrt{2})^2}{(2 + \sqrt{2})^2} = \frac{(2 + \sqrt{2})^2 - (2 - \sqrt{2})^2}{(2 + \sqrt{2})^2}$$

$$\implies a = \frac{(2 + \sqrt{2})^2}{8 \sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{16} (4 + 4\sqrt{2} + 2) \sqrt{2} = \frac{8 + 6\sqrt{2}}{16} = \frac{4 + 3\sqrt{2}}{8} = \underline{\underline{1.03033}}$$

- Pokaži da se energija harmoničkog oscilatora u stanju s kvantnim brojem n može napisati u obliku

$$E_n = m\omega^2 \langle x^2 \rangle \; .$$

#

$$\left[\begin{array}{l}\Psi_n = \psi_n e^{-i\omega_n t} \\ \psi_n = A_n H_n(\xi) e^{-\xi^2/2}, \quad \xi = \sqrt{\frac{m\omega}{\hbar}}x, \quad A_n = \sqrt[4]{\frac{m\omega}{\pi\hbar}} \frac{1}{\sqrt{2^n n!}} \\ \qquad d\xi = \sqrt{\frac{m\omega}{\hbar}}dx\end{array}\right]$$

$$E_n = \hbar\omega(n + 1/2)$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \Psi_n^* x^2 \Psi_n \, dx = \int_{-\infty}^{\infty} \psi_n^* x^2 \psi_n \, dx =$$

$$= \int_{-\infty}^{\infty} A_n^2 H_n^2(\xi) e^{-\xi^2} x^2 \, dx = \left| \begin{array}{l} \xi = \sqrt{\frac{m\omega}{\hbar}}x \\ d\xi = \sqrt{\frac{m\omega}{\hbar}}dx \end{array} \right|$$

$$= \int_{-\infty}^{\infty} A_n^2 H_n^2(\xi) e^{-\xi^2} \frac{\xi^2}{\frac{m\omega}{\hbar}} \frac{d\xi}{\sqrt{\frac{m\omega}{\hbar}}}$$

$$= \left(\frac{\hbar}{m\omega}\right)^{3/2} A_n^2 \int_{-\infty}^{\infty} H_n^2(\xi) e^{-\xi^2} \xi^2 \, d\xi$$

$$= \left| \begin{array}{l} 2\xi H_n(\xi) - 2n H_{n-1}(\xi) = H_{n+1}(\xi) \\ \xi^2 H_n^2(\xi) = \frac{(H_{n+1}(\xi) + 2n H_{n-1}(\xi))^2}{4} \end{array} \right|$$

$$= \frac{1}{4}\left(\frac{\hbar}{m\omega}\right)^{3/2} A_n^2 \left[\int_{-\infty}^{\infty} H_{n+1}^2(\xi) e^{-\xi^2} d\xi + 4n \int_{-\infty}^{\infty} H_{n+1}(\xi) H_{n-1}(\xi) e^{-\xi^2} d\xi \right.$$

$$\left. + 4n^2 \int_{-\infty}^{\infty} H_{n-1}^2(\xi) e^{-\xi^2} d\xi \right] =$$

$$= \frac{1}{4}\left(\frac{\hbar}{m\omega}\right)^{3/2} A_n^2 \left[2^{n+1}(n+1)! \sqrt{\pi} + 0 + 4n^2 \cdot 2^{n-1}(n-1)! \sqrt{\pi} \right]$$

$$= \frac{1}{4}\left(\frac{\hbar}{m\omega}\right)^{3/2} A_n^2 \, 2^{n+1} n! \sqrt{\pi} \left[n+1+n \right]$$

$$= \frac{1}{4}\left(\frac{\hbar}{m\omega}\right)^{3/2} \sqrt{\frac{m\omega}{\pi\hbar}} \frac{1}{\cancel{2^n} \cancel{n!}} 2^{\cancel{n+1}} \cancel{n!} \cancel{\sqrt{\pi}} \left[n+1+n \right]$$

$$= \frac{1}{2} \frac{\hbar}{m\omega} (2n+1) = \frac{\hbar}{m\omega} (n + 1/2)$$

$$E_n = m\omega^2 \langle x^2 \rangle$$

$$= \cancel{m}\omega^2 \frac{\hbar}{\cancel{m}\cancel{\omega}} (n + 1/2)$$

$$= \hbar\omega (n + 1/2)$$