1

$\lambda_{max} = 850\,nm$
$r_B = 4 \cdot 10^{11}\,m$

$P_{TOT} = ?$

$$\lambda_{max} \cdot T = 2.898 \cdot 10^{-3}\,mK$$ WIEN-ov zakon

$$\Rightarrow T = \frac{2.898 \cdot 10^{-3}\,mK}{850 \cdot 10^{-9}\,m} = \underline{3409.41\,K}$$

$$P = \sigma_{SB} \cdot T^4$$ STEFAN-ov zakon

$$\Rightarrow P = 5.67 \cdot 10^{-8}\,W\,m^{-2}\,K^{-4}\,(3409.41\,K)^4$$
$$= \underline{7.66 \cdot 10^{6}\,W/m^2}$$

$$P_{TOT} = P \cdot S = P \cdot 4\pi r^2 = 4\pi\,(4 \cdot 10^{11}\,m)^2 \cdot 7.66 \cdot 10^{6}\,W/m^2$$

$$= \underline{\underline{1.54 \cdot 10^{31}\,W}}$$

2.

$v = 1.88 \cdot 10^6 \, m/s$

$\frac{\Delta v}{v} = 1\% = 0.01$

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a) $\Delta x = ?$

b) $\lambda, \Delta\lambda = ?$

c) $E, \Delta E = ?$

$\frac{v}{c} = \frac{1.88 \cdot 10^6}{3 \cdot 10^8} = 0.0063 \ll 1$

$\Downarrow$

nerelativistička aproksimacija ok

$p = m_e v$

$dp = m_e dv \Rightarrow \Delta p = m_e \Delta v$

$\Delta p = m_e \, 0.01 \cdot v = 0.01 \, p$

a) HEISENBERG-ova relacija neodređenosti

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \geq \frac{\hbar}{2} \frac{1}{\Delta p} = \frac{\hbar}{2} \frac{1}{m_e \Delta v} = \frac{\hbar}{2} \frac{1}{0.01 p}$$

$$m_e = 9.1 \cdot 10^{-31} kg = 0.510 \cdot 10^6 eV/c^2$$

$$p = m_e v = 0{,}510 \cdot 10^6 \frac{eV}{c^2} \cdot 0.0063\, c = 3.2 \cdot 10^3 eV/c$$

$$= 9.1 \cdot 10^{-31} kg \cdot 1.88 \cdot 10^6 m/s$$
$$= 1.71 \cdot 10^{-24} \, kg\, m/s$$

$$\Delta x \geq \frac{hc}{4\pi} \frac{1}{0.01 pc} = \frac{1.240 \cdot 10^{-6} \cancel{eV} m}{4\pi \cdot \underbrace{0.01}_{10^{-2}} \cdot 3.06 \cdot 10^3 \cancel{eV}} = 3.1 \cdot 10^{-9} m$$

$$\Delta x \geq \frac{1.054 \cdot 10^{-34} \overset{kg\frac{m^2}{s^2}\cdot s}{Js}}{2} \frac{1}{10^{-2} \cdot 1.71 \cdot 10^{-24} kg\, m/s} = 3.1 \cdot 10^{-9} m$$

~~b)~~ OUT

de BROGLIE-ova relacija

$$p = \frac{h}{\lambda} \qquad dp = -\frac{h}{\lambda^2} d\lambda \quad \Rightarrow \Delta p = \left| -\frac{h}{\lambda^2} \right| \Delta\lambda$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1.240 \cdot 10^{-6}\, eV\,m}{3.2 \cdot 10^{3}\, eV} = 0.388 \cdot 10^{-9}\, m$$

$$= 0.388\ nm$$

$$= \underline{3.88\ \text{Å}} \quad (\text{x-rays})$$

$$\Delta\lambda = \Delta p \frac{\lambda^2}{h} = 0.01 pc \frac{\lambda^2}{hc} = \frac{0.01 \cdot 3.2 \cdot 10^3\, \cancel{eV}}{1.240 \cdot 10^{-6}\, \cancel{eV\,m}} (3.88 \cdot 10^{-10})^2 m^2$$

$$= \underline{3.8\overset{5}{55} \cdot 10^{-12}\, m}$$

ili

$$\frac{\Delta\lambda}{\lambda} = \frac{3.89 \cdot 10^{-12}\, m}{3.88 \cdot 10^{-10}\, m} \approx 10^{-2}\, m$$

ili

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta p \frac{\lambda^2}{h}}{\frac{h}{p}} = \Delta p \frac{\frac{h^2}{p^2}}{\frac{h}{\frac{h}{p}}} = \Delta p \frac{\cancel{\frac{h^2}{p^2}} \cdot p}{\cancel{h^2}} = \frac{\Delta p}{p}$$

$$= \frac{\Delta v}{v} = 0.01$$

$$\Delta\lambda = 0.01\,\lambda = 3.88 \cdot 10^{-12}\, m$$

~~c)~~ b)

$$E = m_e c^2 + E_k \qquad m_e c^2 = 0.510\ MeV$$

$$E_k = \frac{m_e v^2}{2} = \frac{m_e c^2}{2} \cdot \frac{v^2}{c^2} = \frac{0.510}{2} (0.063)^2\ MeV$$

$$= 10^{-5} \cdot 10^6\, eV = \underline{10\, eV}$$

$$E = \left(0.510 + 0.510 \frac{0.063^2}{2}\right) MeV = (0.510 + 0.00001)\ MeV$$

$$= 0.51001\ MeV$$

V

$$dE_k = \frac{m}{2} 2v\,dv = m v\,dv \quad \Rightarrow \quad \Delta E_k = m v \Delta v$$

$$\frac{\Delta E_k}{E_k} = \frac{m v}{\frac{m v^2}{2}} \Delta v = 2 \frac{\Delta v}{v} = 0.02 \quad \Rightarrow \Delta E_k = 0.02\, E = \underline{0.2\, eV}$$

$E_k \ll m_e c^2$ i.e. $10\,eV \ll 0.51 \cdot 10^6\, eV$

3.

$N = 100\,000$

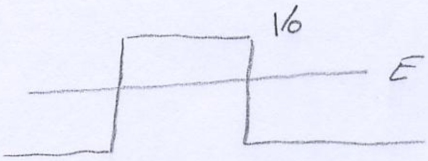
$E = 5\,eV$

$V_0 = 6\,eV$

$a = 0{,}7\,nm$

$\lambda_{dB} = ?$

$N_T = ?$



$\left\{ \begin{array}{l} h = 6.63 \cdot 10^{-34}\,Js \\ c = 2.99 \cdot 10^{8}\,m/s \\ 1eV = 1.6 \cdot 10^{-19}\,CV \end{array} \right.$

$\Downarrow$

$hc = 1.240 \cdot 10^{-6}\,eV\,m$

$m_e c^2 = 0.51 \cdot 10^{6}\,eV$

$E \ll m_e c^2$

$\Downarrow$

nerel. slučaj ✓

$E = \frac{p^2}{2m_e} \Rightarrow$

$\Rightarrow p = \sqrt{2 m_e E}$

$pc = \sqrt{2 m_e c^2 E}$

$= \sqrt{2 \cdot 0{,}51 \cdot 10^{6}\,eV \cdot 5\,eV}$

$= 2258.32\,eV$

$$\lambda_{dB} = \frac{h}{p} = \frac{hc}{p \cdot c}$$

$$= \frac{1{,}240 \cdot 10^{-6}\,eV\,m}{2258{,}32\,eV}$$

$$= 5{,}49 \cdot 10^{-10}\,m$$

$$= \underline{\underline{5{,}49\ \mathring{A}}}$$

$E < V_0$

$$\frac{1}{T} = 1 + \frac{1}{4}\,\frac{V_0^2}{E(V_0 - E)}\,sh^2(\varkappa a)$$

$$\varkappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

$$= \sqrt{\frac{2 m_e c^2 (V_0 - E)}{(\hbar c)^2}}$$

$$= \sqrt{\frac{2 \cdot 0{,}51 \cdot 10^{6}\,\cancel{eV} \cdot 1\,eV}{\left(197{,}33 \cdot 10^{6}\,\cancel{eV} \cdot \frac{fm}{10^{-15}\,m}\right)^2}}$$

$$= \sqrt{\frac{2 \cdot 0{,}51}{197{,}33^2}} \cdot 10^{24}\ m^{-1}$$

$$= \underline{\underline{0{,}51 \cdot 10^{10}\,m^{-1}}}$$

tablice fiz. konst.

$$T = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2\,sh^2(\varkappa a)}$$

$$= \frac{4 \cdot 5 \cdot 1\ \cancel{eV^2}}{4 \cdot 5 \cdot 1\,\cancel{eV^2} + 36\,\cancel{eV^2}\,sh^2(0{,}51 \cdot 10^{10}\,\cancel{m^{-1}} \cdot 0{,}7 \cdot 10^{-9}\,\cancel{m})}$$

$$= \frac{20}{20 + 36\,sh^2(3.57)}$$

$$= 0.0018 = 1{,}8\%$$

$$\Rightarrow N_T = 1800 \text{ elektrona}$$

$\left\{ \begin{array}{l} \hbar c = 197{,}33\ MeV\,fm \\ (\hbar c)^2 = 0.39\ GeV^2\,mbarn \end{array} \right.$

4.

$$\Psi_n = \psi_n e^{-i\omega_n t}$$

$$\psi_n = A_n H_n(\xi) e^{-\xi^2/2}, \quad \xi = \sqrt{\frac{m\omega}{\hbar}}x, \quad A_n = \sqrt[4]{\frac{m\omega}{\pi\hbar}} \frac{1}{\sqrt{2^n n!}}$$

$$d\xi = \sqrt{\frac{m\omega}{\hbar}} dx$$

$$E_n = \hbar\omega(n + 1/2)$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \Psi_n^* x^2 \Psi_n \, dx = \int_{-\infty}^{\infty} \psi_n^* x^2 \psi_n \, dx =$$

$$= \int_{-\infty}^{\infty} A_n^2 H_n^2(\xi) e^{-\xi^2} x^2 \, dx = \left| \begin{array}{l} \xi = \sqrt{\frac{m\omega}{\hbar}} x \\ d\xi = \sqrt{\frac{m\omega}{\hbar}} dx \end{array} \right|$$

$$= \int_{-\infty}^{\infty} A_n^2 H_n^2(\xi) e^{-\xi^2} \frac{\xi^2}{\frac{m\omega}{\hbar}} \frac{d\xi}{\sqrt{\frac{m\omega}{\hbar}}}$$

$$= \left(\frac{\hbar}{m\omega}\right)^{3/2} A_n^2 \int_{-\infty}^{\infty} H_n^2(\xi) e^{-\xi^2} \xi^2 \, d\xi$$

$$= \left| \begin{array}{l} 2\xi H_n(\xi) - 2n H_{n-1}(\xi) = H_{n+1}(\xi) \\ \xi^2 H_n^2(\xi) = \dfrac{(H_{n+1}(\xi) + 2n H_{n-1}(\xi))^2}{4} \end{array} \right|$$

$$= \frac{1}{4}\left(\frac{\hbar}{m\omega}\right)^{3/2} A_n^2 \left[\int_{-\infty}^{\infty} H_{n+1}^2(\xi) e^{-\xi^2} d\xi + 4n \int_{-\infty}^{\infty} H_{n+1}(\xi) H_{n-1}(\xi) e^{-\xi^2} d\xi \right.$$

$$\left. + 4n^2 \int_{-\infty}^{\infty} H_{n-1}^2(\xi) e^{-\xi^2} d\xi \right] =$$

$$= \frac{1}{4}\left(\frac{\hbar}{m\omega}\right)^{3/2} A_n^2 \left[2^{n+1}(n+1)! \sqrt{\pi} + 0 + 4n^2 \cdot 2^{n-1}(n-1)! \sqrt{\pi} \right]$$

$$= \frac{1}{4}\left(\frac{\hbar}{m\omega}\right)^{3/2} A_n^2 \, 2^{n+1} n! \sqrt{\pi} \left[n+1+n \right]$$

$$= \frac{1}{4}\left(\frac{\hbar}{m\omega}\right)^{3/2} \sqrt{\frac{m\omega}{\pi\hbar}} \frac{1}{\cancel{2^n} \cancel{n!}} \cancel{2^{n+1}} \cancel{n!} \cancel{\sqrt{\pi}} \left[n+1+n \right]$$

$$= \frac{1}{2}\,\frac{\hbar}{m\omega}\,(2n+1) = \frac{\hbar}{m\omega}\,(n+1/2)$$

$$E_n = m\omega^2 \langle x^2 \rangle$$

$$= \cancel{m}\,\omega^{\cancel{2}}\,\frac{\hbar}{\cancel{m}\cancel{\omega}}\,(n+1/2)$$

$$= \hbar\omega\,(n+1/2)$$