1.

Ljudsko tijelo

$A = 1.8\,m^2$

$P^{tot} = ?$

$\lambda_{max} = ?$

$\sigma_{SB} = 5.67 \cdot 10^{-8}\ Wm^{-2}K^{-4}$

$$P_T = \sigma_{SB} \cdot T^4$$

(STEFAN-BOLTZMAN-OV zakon)

snaga po jedinici površine crnog tijela

$$P^{tot} = P_T \cdot A$$

$$T = 36\,^\circ C = (36 + 273.15)\,K$$

$$= \underline{309.15\,K}$$

$$P^{tot} = 5.67 \cdot 10^{-8}\ W\cancel{m^{-2}}\cancel{K^{-4}} \cdot (309.15\,\cancel{K})^4 \cdot 1.8\,\cancel{m^2}$$

$$= \underline{\underline{932.25\,W}}$$

$$\lambda_{max} \cdot T = 2.898 \cdot 10^{-3}\,mK$$

(WIEN-OV zakon)

λ_{max}... valna duljina na kojoj je gustoća energije zračenja crnog tijela temp. T maksimalna

$$\lambda_{max} = \frac{2.898 \cdot 10^{-3}\,m\cancel{K}}{309.15\,\cancel{K}} = 9.374 \cdot 10^{-6}\,m$$

$$= \underline{\underline{9.374\,\mu m}}$$

$$= \underline{9374\,nm}$$

IR područje

vidljiva svjetlost 400 – 700 nm

$\lambda_{max} > (400 - 700\,nm)$

$\nu_{max} < \nu_{vidljive\ svj.}$

$E_{max} < E_{vidljive\ svj.}$

2

$\nu = 9{,}5 \cdot 10^{14}\,Hz$

$\phi = 2{,}8\,eV$

a) $U = ?$

b) $\nu_{prag} = ?$

c) $E_k = ?$

$E_{max} = h\nu - \phi$

maksimalna kin. energija elektrona

$\phi = h\nu_{prag}$

a) $E_{max} = eU$

$$\Rightarrow U = \frac{E_{max}}{e} = \frac{h\nu - \phi}{e}$$

$$= \frac{6{.}626 \cdot 10^{-34}\,J\cancel{s}\; 9{.}5 \cdot 10^{14}\,\cancel{s^{-1}} - 2{.}8\,eV}{e}$$

$1\,eV = 1{,}602 \cdot 10^{-19}\,J$

$$= \frac{6{.}626 \cdot 10^{-34} \cdot 9{,}5 \cdot 10^{14} \frac{1}{1{.}602 \cdot 10^{-19}}\,eV - 2{.}8\,eV}{e}$$

$$= \frac{(3{.}93 - 2{.}8)\,\cancel{e}V}{\cancel{e}} = 1{.}13\,V$$

b) $$\nu_{prag} = \frac{\phi}{h} = \frac{\phi c}{hc} = \frac{2{.}8\,\cancel{eV} \cdot c}{1{,}240 \cdot 10^{-6}\,\cancel{eV}\,m}$$

$$= \frac{2{,}8 \cdot 2{.}998 \cdot 10^{8}\,\cancel{m}/s}{1{,}240 \cdot 10^{-6}\,\cancel{m}} = 6{,}77 \cdot 10^{14}\,Hz \;\; s^{-1}$$

c) $E_k \le E_{max} = (3{,}93 - 2{,}8)\,eV = 1{.}13\,eV$

$= 1{,}13 \cdot 1{,}602 \cdot 10^{-19}\,J = 1{,}81 \cdot 10^{-19}\,J$

(3.)

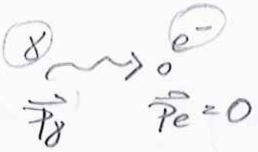
$v^e = 0$

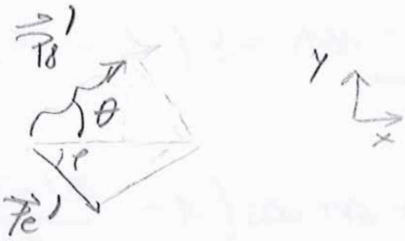
$E_\gamma = 0{,}5\ MeV$

$E_e^{k'} = 0{,}1\ MeV$

$\lambda' = ?$

$\theta = ?$





ZOI:

$$\vec{p}_\gamma = \vec{p}_\gamma\,' + \vec{p}_e\,'$$

x-os $\quad p_\gamma = p_\gamma' \cos\theta + p_e' \cos\varphi$

y-os $\quad 0 = p_\gamma' \sin\theta - p_e' \sin\varphi$

$hc = 1240\ eVnm$

ZOE:

$$E_\gamma + m_e c^2 = E_\gamma' + E_e'$$

$h\nu = h\frac{c}{\lambda}$ $\qquad$ $h\nu' = h\frac{c}{\lambda'}$

$\lambda_c = 2{,}43\ pm$

$$E_e' = m_e c^2 + E_e^{k'}$$

$$\Rightarrow \underbrace{\frac{hc}{\lambda}}_{E_\gamma} - \frac{hc}{\lambda'} = E_e^{k'}$$

$$\Rightarrow \frac{hc}{\lambda'} = E_\gamma - E_e^{k'}$$

$$\Rightarrow \lambda' = \frac{hc}{E_\gamma - E_e^{k'}} = \frac{1240\ \cancel{eV}nm}{(0{,}5 - 0{,}1)\cdot 10^6\ \cancel{eV}}$$

$$= 3100 \cdot 10^{-15} m = 3{,}1 \cdot 10^{-12} m = 3{,}1\ pm$$

$$\lambda' = \lambda + \lambda_c(1 - \cos\theta)$$

$$\Rightarrow \cos\theta = 1 - \frac{\lambda' - \lambda}{\lambda_c}$$

$\Downarrow$

$$\theta = \arccos\left(1 - \frac{\lambda' - \lambda}{\lambda_c}\right)$$

$$= \arccos\left(1 - \frac{3{,}1 - 2{,}48 \ \text{pm}}{2{,}43 \ \text{pm}}\right)$$

$$= \arccos 0{,}745$$

$$= 0{,}73 = 41{,}8255°$$

$$= 41° \, 49' \, 33{,}24''$$

$$\frac{r}{s} = \frac{\pi}{180} \Rightarrow s = \frac{180}{\pi} r$$

$$E_\gamma = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E_\gamma}$$

$$= \frac{1240 \ \cancel{eV} \ nm}{0{,}5 \cdot 10^6 \ \cancel{eV}} \quad (nm = 10^{-9} m)$$

$$= 2480 \cdot 10^{-15} m$$

$$= 2{,}48 \ pm$$

4.

$\lambda_p = \lambda_X$

$E_X = 1\,keV$

a) $v = ?$

b) $\frac{\Delta\lambda}{\lambda} = 10^{-3}$, $\frac{\Delta p}{p} = ?$, $\frac{\Delta v}{v} = ?$, $\frac{\Delta E_k}{E_k} = ?$

c) $\Delta x = ?$

$hc = 1240\,eV\,nm$

$m_p c^2 = 938.27\,MeV$

a)

$\to \gamma \Rightarrow E_X = \frac{hc}{\lambda_X}$ (ν_X)

$\Rightarrow \lambda_X = \frac{hc}{E_X} = \frac{1240\,\cancel{eV}\,nm}{1 \cdot 10^3\,\cancel{eV}}$ ($10^{-9}\,m$)

$= 1240 \cdot 10^{-12}\,m$

$= 1240\,pm = 1.24\,nm$

$\to p: \quad \lambda_p = \lambda_X = 1.24\,nm$

de BROGLIE-ova relacija: $p_p = \frac{h}{\lambda_p}$

$p_p c = \frac{hc}{\lambda_p}$

$p_p c = \frac{1240\,eV\,\cancel{nm}}{1.24\,\cancel{nm}}$

$p_p c = 10^3\,eV = 1\,keV$ ✓

$p_p c \ll m_p c^2$

$\Rightarrow$ nerelativistička aproksimacija je OK

$p_p = m_p v_p$

$\Rightarrow v_p = \frac{p_p}{m_p} \Rightarrow \frac{v_p}{c} = \frac{p_p c}{m_p c^2} = \frac{10^3\,\cancel{eV}}{938.27 \cdot 10^6\,\cancel{eV}}$

$\simeq 10^{-6}$

$\Rightarrow v_p = 10^{-6}\,c = 3 \cdot 10^2\,m/s = 300\,m/s$

b)

$$p = \frac{h}{\lambda}$$

$$dp = -\frac{h}{\lambda^2} d\lambda \Rightarrow \Delta p = \left| -\frac{h}{\lambda^2} \right| \Delta\lambda$$

$$\Delta p = p \frac{\Delta\lambda}{\lambda} \Rightarrow \frac{\Delta p}{p} = 10^{-3} = 0{,}1\%$$

$$\frac{\Delta p}{p} = \frac{\Delta\lambda}{\lambda}$$

$$p = mv$$

$$dp = m\,dv \Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{mv}{m} \frac{\Delta\lambda}{\lambda} = v \frac{\Delta\lambda}{\lambda}$$

$$\Delta v = 300\,m/s \cdot 10^{-3} = 0{,}3\,m/s$$

$$\Rightarrow \frac{\Delta v}{v} = 10^{-3} = 0.1\%$$

$$\frac{\Delta v}{v} = \frac{\Delta\lambda}{\lambda}$$

$$E_k = \frac{mv^2}{2}$$

$$dE_k = \frac{\cancel{2}mv\,dv}{\cancel{2}} \Rightarrow \Delta E_k = mv\Delta v = \frac{mv^2}{2} 2 \frac{\Delta v}{v}$$

$$= 2E_k \frac{\Delta v}{v} = 2E_k \frac{\Delta\lambda}{\lambda}$$

$$\frac{\Delta E_k}{E_k} = 2\frac{\Delta\lambda}{\lambda}$$

$$\Rightarrow \frac{\Delta E_k}{E_k} = 2 \cdot 10^{-3} = 0{,}2\%$$

c)

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$m/\frac{kg\,m}{s^2}\,s$$

$$\Delta x \geq \frac{\hbar}{2} \frac{1}{\Delta p} = \frac{\hbar}{2m\Delta v} = \frac{1.054 \cdot 10^{-34}\,Js}{2 \cdot 1.672 \cdot 10^{-27}\,\cancel{kg}\; 0.3\,\cancel{m}/s}$$

$$= 1.05 \cdot 10^{-7}\,m$$

(5) $He^+ \Rightarrow z=2$

$\lambda = 656{,}5\,nm$

$n \rightarrow n' = 4$

a) $n = ?$

$$h\nu = E_n - E_{n'}$$

$$\frac{hc}{\lambda} = -z^2 \frac{\overset{hcR_H}{R}}{n^2} + z^2 \frac{\overset{hcR_H}{R}}{n'^2}$$

$$\boxed{\frac{1}{\lambda} = z^2 \underset{\frac{4}{b}}{R_H} \left(\frac{1}{n'^2} - \frac{1}{n^2} \right)}$$

$b = 364{,}56\,nm$

$$\frac{1}{\lambda} = \frac{4z^2}{b} \left(\frac{1}{n'^2} - \frac{1}{n^2} \right)$$

$$\frac{1}{n^2} = \frac{1}{n'^2} - \frac{b}{4z^2} \frac{1}{\lambda}$$

$$\Downarrow$$

$$n \approx \sqrt{\frac{4z^2 \lambda n'^2}{4z^2\lambda - n'^2 b}} = \sqrt{\frac{\cancel{4 \cdot 4} \cdot 656{,}5\,\cancel{nm} \cdot \cancel{16}}{\cancel{4 \cdot 4} \cdot 656{,}5\,\cancel{nm} - \cancel{16}\ 364{,}56\,\cancel{nm}}}$$

$$= 4 \sqrt{\frac{656{,}5}{656{,}5 - 364{,}56}} \approx 6$$

b) $e \to \pi^-$, $m_\pi = 274\, m_e$

$\lambda = ?$

$m_p = 1.836 \cdot 10^3\, m_e$

He^+ → stabilni izotop $^4He^+$ $\Rightarrow m_{jezgre} = 4\, m_p$ $\left\{ \begin{array}{l} 2p \\ 2n \end{array} \right.$ } tzv. α-čestica

reducirana masa: $\mu = \frac{m_1 m_2}{m_1 + m_2}$

$$\mu = \frac{(4m_p)(274\, m_e)}{4m_p + 274\, m_e} = \frac{4 \cdot 1836\, \cancel{m_e} \cdot 274\, m_e}{4 \cdot 1836\, \cancel{m_e} + 274\, \cancel{m_e}} = 264.15 \approx 264\, m_e$$

$$\approx 0{,}96\, m_\pi$$

$$R_H' = R_H \frac{\overset{\mu}{264\, m_e}}{m_e} = 264\, R_H$$

$\frac{1}{\lambda}$ iz a) dijela zadatka

$$\frac{1}{\lambda'} = Z^2 R_H' \left(\frac{1}{n'^2} - \frac{1}{n^2} \right) = 264\, Z^2 R_H \left(\frac{1}{n'^2} - \frac{1}{n^2} \right)$$

$$\Rightarrow \lambda' = \frac{1}{264} \lambda = \frac{1}{264}\, 656.5\, nm = 2{,}49\, nm$$

za $\mu = \frac{(4m_p) \cdot m_\pi}{4m_p + m_\pi} = \frac{m_\pi}{1 + \underbrace{\frac{m_\pi}{4m_p}}_{\text{zanemarimo}}} \approx m_\pi$

$$R_H'' = R_H \frac{\overset{\mu}{m_\pi}}{m_e} = R_H \frac{274\, m_e}{m_e} = 274\, R_H$$

$$\lambda'' = \frac{1}{274}\, 656.5\, nm = 2.40\, nm$$

$$\frac{\lambda''}{\lambda'} = \frac{2.40}{2.49} = \frac{\frac{1}{274}}{\frac{1}{264}} \approx 0.96 \Rightarrow \text{greška } 4\%$$

alternativno, He^+ → nestabilni (vrlo rijetki) izotop $^2He^+$ $\Rightarrow m_{jezgre} = 2\, m_p$ $\{ 2p$

$$\mu = \frac{(2m_p)(274\, m_e)}{2m_p + 274\, m_e} = 254{,}97\, m_e \approx 255\, m_e \approx 0{,}93\, m_\pi$$

$$R_H''' = R_H \cdot 255 \Rightarrow \lambda''' = \frac{1}{255} \lambda = 2{,}57\, nm$$

$$\frac{\lambda''}{\lambda'''} = \frac{2.40}{2{,}57} \approx 0{,}93 \Rightarrow \text{greške } 7\%$$

Teorija

1. $V(x,t) = V(x)$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + \underline{\underline{V(x)}}\,\psi(x,t) = i\hbar\frac{\partial \psi(x,t)}{\partial t}$$ vremenski ovisna Sch. jednadžba

$$\psi(x,t) = \psi(x)\,\varphi(t)$$ separacija varijabli

$$-\frac{\hbar^2}{2m}\varphi(t)\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x)\varphi(t) = i\hbar\psi(x)\frac{\partial \varphi(t)}{\partial t} \quad /:\psi(x)\varphi(t)$$

$$\underbrace{-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)}_{\text{prostorno ovisan dio}} = \underbrace{i\hbar\frac{1}{\varphi(t)}\frac{\partial \varphi(t)}{\partial t}}_{\text{vremenski ovisan dio}} = E$$

$$\rightarrow \boxed{i\hbar\frac{1}{\varphi(t)}\frac{\partial \varphi(t)}{\partial t} = E}$$

$$\frac{d\varphi(t)}{dt} = -i\frac{E}{\hbar}\varphi(t) \longrightarrow \frac{d\varphi}{\varphi} = -i\frac{E}{\hbar}dt \quad /\int$$

$$\ln\varphi = -i\frac{E}{\hbar}$$

$$\varphi(t) = e^{-i\frac{E}{\hbar}t}$$

$$\rightarrow \boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)}$$ vremenski neovisna Sch. jednadžba

$$\psi(x,t) = \psi(x)\,e^{-i\overbrace{\frac{E}{\hbar}}^{\omega}t}$$

$\{\ E = \hbar\omega$

2

a) $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

Svojstveno stanje operatora energije $\Rightarrow \Delta E = 0$

$\Downarrow$

$\Delta t = \infty$

i $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

b) Da.

c)

$t=0 \qquad \varphi_E(x)$

$t \qquad \underbrace{\varphi_E(x)\, e^{-i\omega t}}_{\psi(x,t)} \qquad \omega = \frac{E}{\hbar}$

d)

$$P(x,t) = \psi^* \psi = \varphi_E^*(x)\, e^{i\omega t}\, \varphi_E(x)\, e^{-i\omega t}$$

$$= |\varphi_E(x)|^2 \quad \{ \neq f(t)$$

3.

a) PLANCKov zakon zračenja

$$\boxed{u_T(\nu) = \frac{8\pi\nu^2}{c^3}\,\frac{h\nu}{e^{h\nu/kT}-1}}$$

vrijedi $\forall \nu$

RAYLEIGH-JEANS-ov zakon

$$\boxed{u_T^{(RJ)}(\nu) = \frac{8\pi\nu^2}{c^3}\,kT}$$

vrijedi za $\nu \ll$ tj. $\nu \ll \frac{kT}{h}$

$\frac{h\nu}{kT} \ll 1$

Izvod:

$$u_T(\nu) \xrightarrow[\frac{h\nu}{kT}\ll]{} \frac{8\pi h\nu^3}{c^3}\,\frac{1}{\cancel{1}+\frac{h\nu}{kT}-\cancel{1}} = \frac{8\pi \cancel{h}\nu^{\cancel{3}\,2}}{c^3}\,\frac{kT}{\cancel{h\nu}}$$

$$= \frac{8\pi\nu^2}{c^3}\,kT$$

WIEN-ova aproksimacija

$$\boxed{u_T^{W}(\nu) = \frac{8\pi h\nu^3}{c^3}\,e^{-\frac{h\nu}{kT}}}$$

vrijedi za $\nu \gg$ tj. $\nu \gg \frac{kT}{h}$

$\frac{h\nu}{kT} \gg 1$

Izvod:

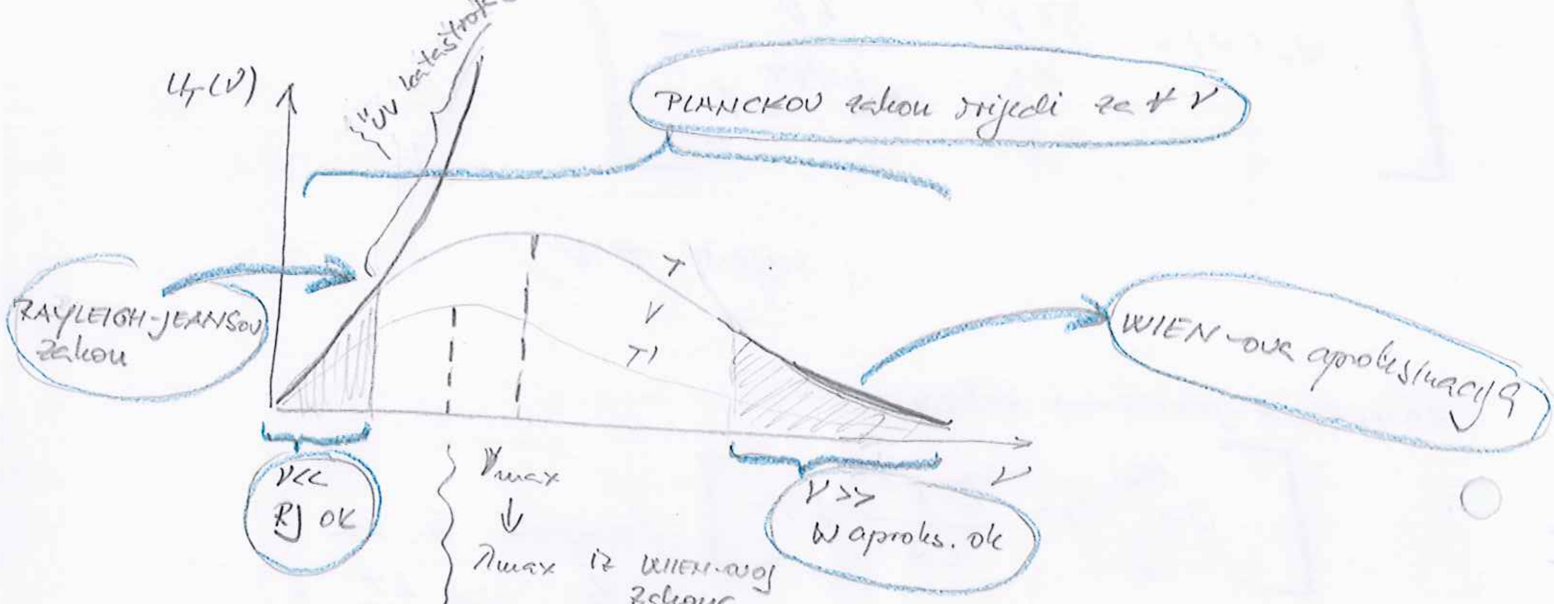
$$u_T(\nu) \xrightarrow[\frac{h\nu}{kT}\gg]{} \frac{8\pi h\nu^3}{c^3}\,e^{-\frac{h\nu}{kT}}$$

$\Downarrow\ e^{\frac{h\nu}{kT}} \gg 1$

$\Downarrow\ e^{\frac{h\nu}{kT}}-1 \approx e^{h\nu/kT}$

b)

$\boxed{\begin{array}{ll} \text{RJ zakon} & \nu \ll \\ \text{W aproksimacija} & \nu \gg \end{array}}$



ν_{max}

$\Downarrow$

λ_{max} iz WIEN-ovog zakona

$\lambda_{max} \cdot T = 2.898 \cdot 10^{-3}\,mK$