# 1. kolokvij (23. 4. 2015.)

**1.**

$2r = 1\ mm$

$l = 5\ cm$

$P = 10\ W$

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$T = ?$

Stefan–Boltzmanov zakon:

$$\left[ \frac{P}{A} = \sigma_{SB} T^4 \right]$$

$\sigma_{SB} = 5.67 \cdot 10^{-8}\ W m^{-2} K^{-4}$

$$\Rightarrow T = \sqrt[4]{\frac{P}{A} \frac{1}{\sigma_{SB}}}$$

$$= \sqrt[4]{\frac{10\ W}{1.57 \cdot 10^{-4}\ m^2} \frac{1}{5.67 \cdot 10^{-8}\ W m^{-2} K^{-4}}}$$

$$= \sqrt[4]{\underbrace{\frac{1}{1.57}}_{0.637} \cdot \underbrace{\frac{1}{5.67}}_{0.176} 10^{13}}\ K$$

(0.113)

Mathematica: $\underline{1029.5\ K}$ (1024.7 K)

$$= \sqrt[4]{1.12 \cdot 10^{12}}\ K$$

$$= 1.029 \cdot 10^3\ K = \underline{\underline{1029\ K}}$$

žarna nit → valjak

$A = 2r\pi \cdot l$

$= 10^{-3}\ m\ \pi \cdot 5 \cdot 10^{-2}\ m$

$= 1.57 \cdot 10^{-4}\ m^2$

baza $B = r^2 \pi$

$= (0.5 \cdot 10^{-3})^2 \pi$

$= 7.85 \cdot 10^{-7}\ m^2$

$2B = 1.57 \cdot 10^{-6}\ m^2$

— zanemarivo

(2.)

$\lambda = 4500 Å = 4500 \cdot 10^{-10} m$

$U = 0.75 V$

$\lambda' = 3000 Å$

$U' = ?$

**Fotoefekt**

$$\left[ \begin{array}{l} E_{max} = h\nu - \phi = eU \\ E'_{max} = h\nu' - \phi = eU' \end{array} \right] \quad \nu\lambda = c$$

$$\frac{hc}{\lambda} - \phi = eU \quad \Rightarrow \phi = \frac{hc}{\lambda} - eU$$

$$\frac{hc}{\lambda'} - \phi = eU'$$

$$\left[ eU' = \frac{hc}{\lambda'} - \frac{hc}{\lambda} + eU \right]$$

$$hc = 6.626 \cdot 10^{-34} Js \cdot 2.998 \cdot 10^{8} m/s$$

$$= 1.986 \cdot 10^{-25} Jm = \quad / 1eV = 1.602 \cdot 10^{-19} J /$$

$$= \frac{1.986 \cdot 10^{-25}}{1.602 \cdot 10^{-19}} eVm$$

$$= 1.240 \cdot 10^{-6} eVm$$

$$eU' = \frac{12.40 \cdot 10^{-7} eVm}{3 \cdot 10^{-7} m} - \frac{12.40 \cdot 10^{-7} eVm}{4.5 \cdot 10^{-7} m} + 0.75 eV$$

$$= (4.133 - 2.756 + 0.75) eV$$

$$= 2.127 eV \quad \rightarrow \quad U' = 2.13 V$$

(3)

$\lambda = 1\ pm$

$\theta = 90^0$

$E_\gamma - E_\gamma' = ?$

$\lambda_e' = ?$


COMPTON-ovo raspršenje:

$$\left[\ \lambda' = \lambda + \lambda_c (1 - \cos\theta)\ \right] \qquad \lambda_c = 2.43 \cdot 10^{-12} m$$

$$\lambda' = 10^{-12} + 2.43 \cdot 10^{-12} (1 - 0)$$

$$= 3.43 \cdot 10^{-12}\ m$$

$$E_\gamma = h\nu = \frac{hc}{\lambda} = \frac{1,240 \cdot 10^{-6}\ eVm}{10^{-12}\ m} = 1,240 \cdot 10^6\ eV$$

$\{ = 1.99 \cdot 10^{-13} J$

$$E_\gamma' = h\nu' = \frac{hc}{\lambda'} = \frac{1,240 \cdot 10^{-6}\ eVm}{3.43 \cdot 10^{-12} m} = 0,36 \cdot 10^6\ eV$$

$\{ = 0.58 \cdot 10^{-13} J$

$\Downarrow$

$$E_\gamma - E_\gamma' = (1,240 - 0,36) \cdot 10^6\ eV = 0.88 \cdot 10^6\ eV$$

$\{ = 1.41 \cdot 10^{-13} J$

ZOE:

$$E_\gamma + m_e c^2 = E_\gamma' + \underbrace{m_e c^2 + E_k}_{E_e'}$$

$$E_e' = E_\gamma - E_\gamma' + m_e c^2 = 0.88 \cdot 10^6 eV + 0,510 \cdot 10^6 eV$$

$$= 1.39 \cdot 10^6\ eV$$

de-BROGLIE-ova relacija

$$p_e = \frac{h}{\lambda_e} \Rightarrow \lambda_e = \frac{hc}{p_e c} = \frac{1.240 \cdot 10^{-6} eV m}{1,29\cancel{3} \cdot 10^6 eV} = 0.96 \cdot 10^{-12} m$$

$= 0.96 pm$

$\rightarrow E_e^2 = (m_e c^2)^2 + p_e^2 c^2$ (relativistička rel. između E i P)

$$\Rightarrow p_e c = \sqrt{E_e^2 - (m_e c^2)^2} = \sqrt{1,39^2 - 0,51^2} \cdot 10^6\ eV = 1.29 \cdot 10^6\ eV$$

$\{ p = 6,89 \cdot 10^{-22}\ kg\ m/s$

$= 4.3 \cdot 10^{-3}\ eVs/m$

(4.)

$v = 1.88 \cdot 10^6 \, m/s$

$\frac{\Delta v}{v} = 1\% = 0.01$

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a) $\Delta x = ?$

b) $\lambda, \Delta\lambda = ?$

c) $E, \Delta E = ?$

$\frac{v}{c} = \frac{1.88 \cdot 10^6}{3 \cdot 10^8} = 0.0063 \ll 1$

⇓

nerelativističke aproksimacija OK

$p = m_e v$

$dp = m_e dv \Rightarrow \Delta p = m_e \Delta v$

$\Delta p = m_e \, 0.01 \cdot v = 0.01 \, p$

a) HEISENBERG-ova relacija neodređenosti

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \geq \frac{\hbar}{2} \frac{1}{\Delta p} = \frac{\hbar}{2} \frac{1}{m_e \Delta v} = \frac{\hbar}{2} \frac{1}{0.01 p}$$

$$m_e = 9.1 \cdot 10^{-31} kg = 0.510 \cdot 10^6 eV/c^2$$

$$p = m_e v = 0.510 \cdot 10^6 \frac{eV}{c^2} \cdot 0.0063 \, c = 3.2 \cdot 10^3 eV/c$$

$$= 9.1 \cdot 10^{-31} kg \cdot 1.88 \cdot 10^6 m/s$$

$$= 1.71 \cdot 10^{-24} \, kg\,m/s$$

$$\Delta x \geq \frac{hc}{4\pi} \frac{1}{0.01 pc} = \frac{1.240 \cdot 10^{-6} \, eVm}{4\pi \cdot \underbrace{0.01}_{10^{-2}} \cdot 3.06 \cdot 10^3 eV} = 3.1 \cdot 10^{-9} m$$

$$\Delta x \geq \frac{1.054 \cdot 10^{-34} Js}{2} \frac{1}{10^{-2} \cdot 1.71 \cdot 10^{-24} kg\,m/s} = 3.1 \cdot 10^{-9} m \checkmark$$

($Js = kg \frac{m^2}{s^2} s$)

~~b)~~ OUT

de BROGLIE-ova relacija

$$p = \frac{h}{\lambda} \qquad dp = -\frac{h}{\lambda^2}d\lambda \quad \Rightarrow \Delta p = \left|-\frac{h}{\lambda^2}\right|\Delta\lambda$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1.240 \cdot 10^{-6}\, eV\, m}{3.2 \cdot 10^3\, eV} = 0.388 \cdot 10^{-9}\, m$$

$$= 0.388\ nm$$

$$= \underline{\underline{3.88\ \text{Å}}} \quad (\text{x-rays})$$

$$\Delta\lambda = \Delta p \frac{\lambda^2}{h} = 0.01 pc \frac{\lambda^2}{hc} = \frac{0.01 \cdot 3.2 \cdot 10^3\, eV}{1.240 \cdot 10^{-6}\, eV\, m}(3.88 \cdot 10^{-10})^2 m^2$$

$$= \underline{\underline{3.885 \cdot 10^{-12}\, m}}$$

ili

$$\frac{\Delta\lambda}{\lambda} = \frac{3.89 \cdot 10^{-12}\, m}{3.88 \cdot 10^{-10}\, m} \approx 10^{-2}\, m$$

ili

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta p \frac{\lambda^2}{h}}{\frac{h}{p}} = \Delta p \frac{\frac{h^2}{p^2}\frac{1}{h}}{\frac{h}{p}} = \Delta p \frac{\frac{h^2}{p^2}\cdot p}{h^2} = \frac{\Delta p}{p}$$

$$= \frac{\Delta v}{v} = 0.01$$

$$\Delta\lambda = 0.01\lambda = 3.88 \cdot 10^{-12}\, m$$

~~c)~~ b)

$$E = m_e c^2 + E_k \qquad m_e c^2 = 0.510\ MeV$$

$$E_k = \frac{m_e v^2}{2} = \frac{m_e c^2}{2} \cdot \frac{v^2}{c^2} = \frac{0.510}{2}(0.063)^2\ MeV$$

$$= 10^{-5} \cdot 10^6\, eV = \underline{\underline{10\, eV}}$$

$$E = \left(0.510 + 0.510\frac{0.063^2}{2}\right) MeV = (0.510 + 0.00001)\ MeV$$

$$= 0.51001\ MeV$$

$$dE_k = \frac{m}{2} 2v\,dv = m v\, dv \quad \Rightarrow \Delta E_k = m v \Delta v$$

$$\frac{\Delta E_k}{E_k} = \frac{m v}{\frac{m v^2}{2}}\Delta v = 2\frac{\Delta v}{v} = 0.02 \quad \Rightarrow \Delta E_k = 0.02\, E = \underline{\underline{0.2\, eV}}$$

$$E_k \ll m_e c^2 \quad \text{i.e.} \quad 10\, eV \ll 0.51 \cdot 10^6\, eV$$

(5)

Lymanova serija : $m=1$ , $n>m$

Paschenova serija : $m=3$ , $n>m$

a) $m=1$ $n=2$ ? $n=\infty$ ?

$m=3$ $n=4$ ? $n=\infty$ ?

b) $Li^{2+}$ $(Z=3)$ $m=1, n=2$ ? $n=\infty$ ?

$m=3, n=4$ ? $n=\infty$ ?

ionizacijska energ. ?

a) RYDBERG-ova formula

$$\frac{1}{\lambda_{mn}} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda_{mn}} = \frac{4}{b} \frac{n^2 - m^2}{m^2 n^2}$$

$$\left[\lambda_{mn} = \frac{b}{4} \frac{m^2 n^2}{n^2 - m^2}\right] = \frac{b}{4} \frac{m^2}{1 - m^2/n^2}$$

$\frac{b}{4}$: $1/R_H$

$b = 364.56$ nm

$m=1$ (Lyman)

$$\lambda_{1n} = \frac{b}{4} \frac{n^2}{n^2 - 1} = \frac{364.56\,\text{nm}}{4} \frac{n^2}{n^2-1}$$

($\frac{364.56\,\text{nm}}{4}$ = 91.16 nm)

$n=2$

$$\lambda_{2n} = 91.16\,\text{nm} \frac{4}{4-1} = \frac{4}{3} 91.16\,\text{nm} = 121.55\,\text{nm}$$

$n=\infty$

$$\lambda_{2\infty} = 91.16\,\text{nm}$$

$m=3$ (Paschen)

$$\lambda_{3n} = \frac{b}{4} \frac{9n^2}{n^2 - 9} = \frac{b}{4} \cdot 9 \frac{1}{1 - 9/n^2}$$

($\frac{b}{4} \cdot 9$ = 820.46 nm)

$n=4$

$$\lambda_{34} = 820.46\,\text{nm} \frac{1}{1 - 9/16} = \frac{16}{7} 820.46\,\text{nm} = 1875.34\,\text{nm}$$

$n=\infty$

$$\lambda_{3\infty} = 820.46\,\text{nm}$$

b) $Li^{2+}$ $(Z=3)$

$$\left[ E_n = -Z^2 \frac{R}{n^2} \right]$$

$$h\nu = E_n - E_m = \frac{hc}{\lambda} = \left(-Z^2 \frac{R}{n^2}\right) + Z^2 \frac{R}{m^2}$$

$R_H = 4/b$

$$\frac{1}{\lambda_{mn}} = Z^2 \frac{R}{hc} \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\Rightarrow \quad R_H \rightarrow Z^2 R_H$$

$$\left[ \lambda_{mn}^{(Z)} = \frac{1}{Z^2 R_H} \frac{m^2 n^2}{m^2 - n^2} = \frac{1}{Z^2} \frac{b}{4} m^2 \frac{1}{1 - m^2/n^2} \right]$$

$Li^{2+}$, $Z=3$ $\Rightarrow$ $\lambda_{mn}^{(3)} = \frac{1}{9} \lambda_{mn}^{(1)} = \frac{1}{9} \lambda_{mn}^{(\text{vodik})}$ iz prošlog dijela zadatka

$\Downarrow$

$m=1$

$n=2$ $\quad \lambda_{12}^{(3)} = 13.51\,nm$

$n=\infty$ $\quad \lambda_{1\infty}^{(3)} = 10.13\,nm$

$m=3$

$n=4$ $\quad \lambda_{34}^{(3)} = 208.37\,nm$

$n=\infty$ $\quad \lambda_{3\infty}^{(3)} = 91.16\,nm$

ionizacijska energija: $-E_1 = Z^2 R = 9 \cdot 13.6\,eV = 122.4\ eV$

# Teorija

## 1.

$\mu^-\mu^+$:

⊙ $\left[ E_n = -\frac{R'}{n^2} \right]$

$R' = R(m_e \to \mu) = \left| \mu = \frac{m_\mu m_\mu}{m_\mu + m_\mu} = \frac{m_\mu}{2} = \frac{207 m_e}{2} \right|$ !

$= k_c^2 \frac{m_\mu/2\, e^4}{2\hbar} = \frac{m_\mu/2}{m_e} R = \frac{207}{2} R$

$\left[ r_n = n^2 a_0' \right]$

$a_0' = a_0 (m_e \to \frac{m_\mu}{2})$

$= \frac{1}{k_c} \frac{\hbar^2}{m_\mu/2\, e^2} = \frac{m_e}{m_\mu/2} a_0 = \frac{2}{207} a_0$

$v_n = \frac{v_1}{n}$ isto

⊙ $R' = \frac{207}{2} R \implies \frac{1}{\lambda_{mn}} = \frac{R'}{hc} \frac{n^2 - m^2}{m^2 n^2}$

$\lambda_{mn} = \frac{hc}{R'} m^2 \frac{n^2}{n^2 - m^2}$

$\lambda_{mn} = \frac{2}{207} \frac{hc}{R} m^2 \frac{n^2}{n^2 - m^2}$

BALMER $(m=2)$

$\lambda_{2n} = \frac{2}{207} \underbrace{\frac{hc}{R} \cdot 4}_{b} \frac{n^2}{n^2 - 2^2}$, $n > 2$

$b = 364.56\,nm$

$\lambda_{23} = \frac{2}{207} \underbrace{364.56 \frac{9}{9-4}}_{656.3\,nm} = 6.34\,nm$

→ nije vidljivi dio spektra već X-zrake

(2.)

**a)** $\hat{p}$ : ravni val $\varphi(x) = A e^{ikx} + B e^{-ikx}$

$\hat{x}$ : delta funkcija $\varphi(x) = \delta(x)$

$$\hat{p}_x \varphi(x) = -i\hbar \frac{\partial}{\partial x}\varphi = p_x \varphi$$

$$\hat{x}\varphi(x) = x\varphi(x)$$

**b)**

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

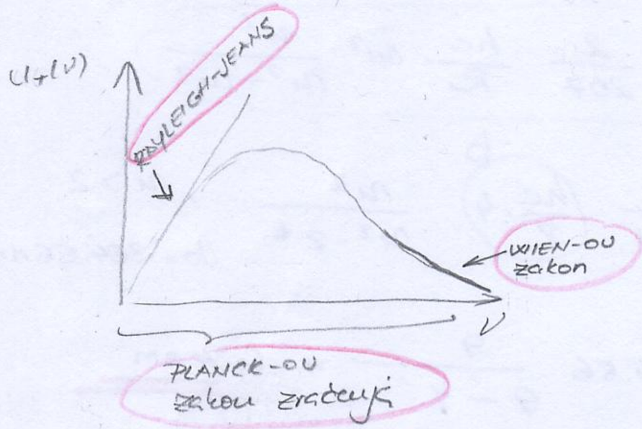
$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

---> nemogućnost određivanja položaja i impulsa/brzine čestice do proizvoljno male točnosti itd.

**c)** Princip korespodencije: Klasično ponašanje se dobiva u limesu velikih kvantnih brojeva $n \to \infty$.

Princip komplementarnosti: Valna i čestična priroda materije se ne mogu naći u istom trenutku iako jedino oba opisa uzeta zajedno omogućuje potpuni uvid u prirodu materije.

(3.)




$$u_T(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/k_BT}-1}$$

RAYLEIGH-JEANS-OV zakon

$\nu \ll k_BT/h$ i.e. $h\nu \ll k_BT$, $\frac{h\nu}{k_BT} \ll 1$

$$e^{h\nu/kT} \approx 1 + \frac{h\nu}{kT} + \dots$$

$$u_T(\nu) \to \frac{8\pi\nu^2}{c^3}\frac{h\nu}{1+\frac{h\nu}{kT}+\dots-1} = \frac{8\pi\nu^2}{c^2}kT$$

WIEN-OV zakon

$\nu \gg k_BT/h$, i.e. $h\nu \gg k_BT$, $\frac{h\nu}{k_BT} \gg 1$

$$e^{\frac{h\nu}{k_BT}} \gg 1$$

$$u_T(\nu) \to \frac{8\pi\nu^2}{c^3}h\nu e^{-h\nu/k_BT}$$